

# Chapter 8

## Taxation

### 8.1 Public Good Financing

There are some goods that it is unreasonable—or even dangerous—to expect the private sector to produce. Such goods are known as public goods; they have two defining characteristics:

1. non-rival consumption
2. non-excludability

The consumption of a good is non-rival if one consumer's consumption of it doesn't affect any other consumer's consumption of it. Consider, for example, a cable television program. The fact that you watch it has no affect on my consumption of it.

The consumption of a good is non-excludable if, once the good has been produced, it is impossible (or unfeasible) to exclude consumers from consuming it. Consider, for example, clean air. Once a nation's air is clean, there is no way to prevent consumers from consuming it. Notice that clean air is also non-rival. Your consumption of it has no affect upon my consumption of it. Thus, clean air is a public good. A cable television program is not a public good, since the cable company can exclude me from consuming it.

Public goods pose a problem because consumers won't be willing to pay to consume them because of the free-rider problem: everyone is tempted to not pay and hope that someone else does. Public goods are therefore usually provided by the government. To pay for public goods, governments must raise money.

The most common method by which the government raises money is taxation. There are all sorts of taxes—income tax, sales tax, capital gains tax, etc.—but they all have one thing in common: they give the government the revenue it needs to provide public goods. In this chapter I'll examine one specific type of tax and see its effect on producers and consumers. I'll then show that the resulting equilibrium is inefficient and examine possible solutions to the inefficiency created by taxation.

## 8.2 A Sales Tax

Assume that there are two goods in our economy:  $T$  and  $U$ . To raise money, the government introduces a sales tax  $t$  on good  $T$ . After the tax is introduced, consumers pay  $P_T^C$  for each unit of the good  $T$  that they purchase. Firms receive  $P_T^F$  for each unit sold of the good that they sell. The two prices are linked as follows:

$$P_T^C = P_T^F(1 + t) = P_T^F + tP_T^F \quad (8.1)$$

For each unit of output of good  $T$  that is sold, the government earns  $tP_T^F$  in tax revenue. Notice that the tax means that consumers and producers react to different prices: consumers choose their consumption bundle based on  $P_T^C$  while producers choose their output quantity based on  $P_T^F$ . Notice also that as the price of good  $T$  rises, the difference between the prices— $tP_T^F$ —increases, as does the government's revenue per unit sold—also  $tP_T^F$ .

By assumption, good  $N$  is not taxed, so consumers and firms react to the same price:

$$P_U^F = P_U^C \quad (8.2)$$

## 8.3 Supply and Demand

Figure 8.1 illustrates demand and supply for good  $T$ . Consumers choose their demand  $D(P_T^C)$  based on  $P_T^C$ . Firms choose their output quantity  $S(P_T^F)$  based on  $P_T^F$ . The intersection of demand and supply is not an equilibrium because this would imply that  $P_T^C = P_T^F$ , which we know to be false.

To combine demand and supply, I must represent both as functions of the same price. Figure 8.2 shows supply as a function of  $P_T^F$  and of  $P_T^C$ .

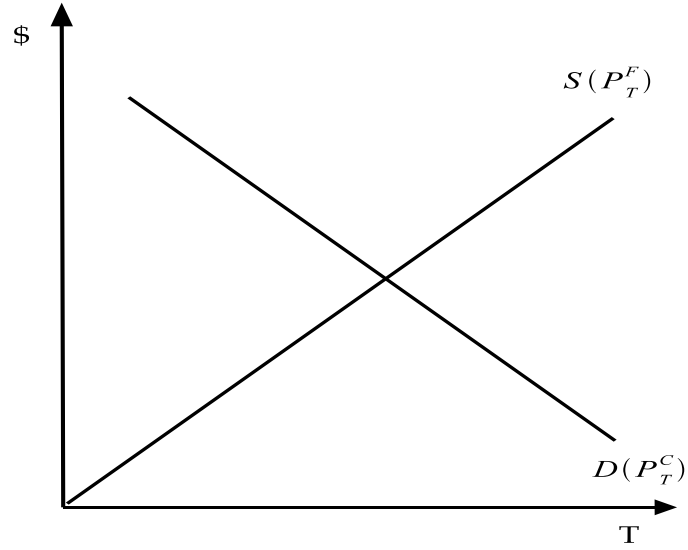


Figure 8.1: Incompatible Demand and Supply

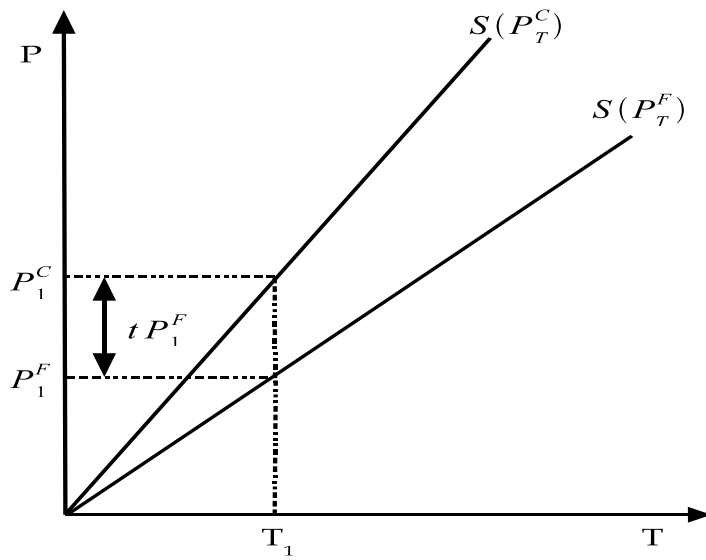


Figure 8.2: Deriving a Compatible Supply Curve

Figure 8.2 also shows how I derived  $S(P_T^C)$  from  $S(P_T^F)$ . The first step is to pick a producer price such as  $P_1^F$  (in this paragraph I drop the  $T$  subscript for specific prices to improve legibility). At this price the height of the supply curve is the total output level in the industry,  $T_1$ . To find the consumer price that will induce the industry to produce  $T_1$ , I rely upon the relationship between the producer price and the consumer price: if the producer receives  $P_1^F$  the consumer must be paying  $P_1^F + tP_1^F$ . Thus,  $P_1^C = P_1^F + tP_1^F$ , and the height of the supply curve  $S(P_T^C)$  for the price  $P_1^F$  is the height of the supply curve  $S(P_T^F)$  plus  $tP_1^F$ .

This is a general rule: the height of the supply curve  $S(P_T^C)$  is the height of the supply curve  $S(P_T^F)$  plus  $tP_T^F$ . This general relationship allows me to plot the supply curve  $S(P_T^C)$  using the supply curve  $S(P_T^F)$ . Notice that the difference between the two curves increases as the price of good  $T$  rises, since the size of  $tP_T^F$  increases.

Figure 8.3 shows the demand and supply curves for  $P_T^C$ . The equilibrium output level  $T^*$  occurs at their intersection. At  $T^*$ , consumers pay  $P_T^{C*}$  and firms receive  $P_T^{F*}$ . The government receives  $tP_T^{F*}$  in tax revenue per unit sold. Total tax revenue is the box  $P_T^{F*}-P_T^{C*}-B-C$ : the tax per unit of output  $tP_T^{F*}$  multiplied by the output level  $T^*$ . Total revenue for the firms in the industry is the box  $A-P_T^{F*}-C-T^*$ :  $P_T^{F*}$  multiplied by the output level  $T^*$ .

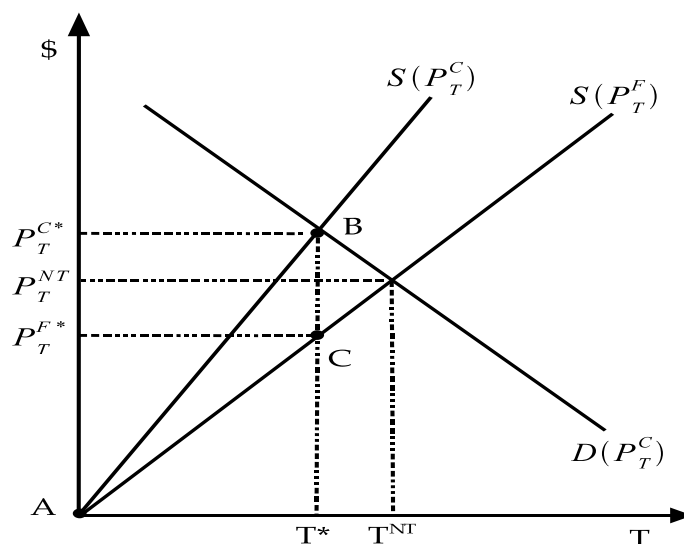


Figure 8.3: Supply and Demand with Taxation

Figure 8.3 also shows the outcome if there was no tax on good  $T$ . If there is no tax then the vertical distance between  $S(P_T^F)$  and  $S(P_T^C)$  is equal to zero (since  $t$  equals zero,  $tP_T^F$  equals zero. Thus, the supply curve  $S(P_T^C)$  is simply the supply curve  $S(P_T^F)$ —consumer and firms are facing the same price—and the equilibrium point is the intersection between  $S(P_T^F)$  and  $D(P_T^C)$ . The equilibrium output level without taxation  $T^{NT}$  is larger than  $T^*$ , consumers pay a lower price ( $P_T^{NT} < P_T^{C*}$ ), and firms receive a higher price ( $P_T^{NT} > P_T^{F*}$ ).

## 8.4 Efficiency

Assume that industries  $T$  and  $U$  are perfectly competitive and that all consumers face the same prices. If all consumers face the same prices, the slope of every consumer's budget constraint is

$$-\frac{P_U^C}{P_T^C} \quad (8.3)$$

This is therefore also equal to the slope of the consumers' indifference curves at their points of consumption. If firms are perfectly competitive, the slope of the PPF is equal to the ratio of the marginal costs in industries  $T$  and  $U$ . At the point of production, this ratio is in turn equal to the ratio of the prices that firms receive in industries  $T$  and  $U$ .

$$\frac{\Delta T}{\Delta U} = -\frac{MC_U}{MC_T} = -\frac{P_U^F}{P_T^F} \quad (8.4)$$

The equilibrium will be efficient if the slope of the consumers' indifference curves at their points of consumption is equal to the slope of the PPF at the point of production. That is, the equilibrium is efficient if (after canceling the minus signs)

$$\frac{P_U^C}{P_T^C} = \frac{P_U^F}{P_T^F} \quad (8.5)$$

The numerators in equation 8.5 are equal, but the denominators are not equal because  $P_T^C > P_T^F$ . Thus equation 8.5 is not satisfied and the equilibrium is therefore inefficient. Figure 8.4 depicts the situation.

Consumers believe good  $T$  to be more costly to produce than it really is so they demand too little of it. The result is an equilibrium in which

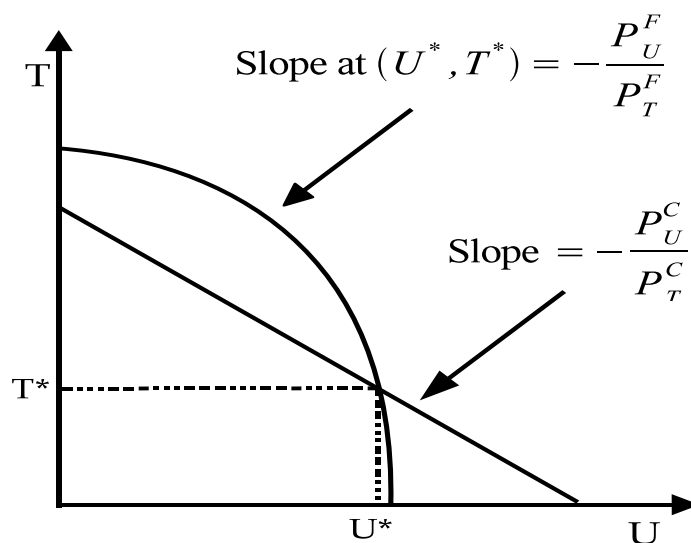


Figure 8.4: Inefficiency due to Taxation of One Good

too much  $U$  and too little  $T$  is produced. Introducing a tax has distorted the price system so that it no longer accurately reflects marginal costs. The result is an inefficient equilibrium.

If the equilibrium is inefficient, there must be a trade that makes someone better off without harming anyone else. Figure 8.5 illustrates one such possible trade. A consumer who is willing to pay at most  $P_\delta$  for one unit of good  $T$  can purchase the good for  $P_\beta$  if both the consumer and the firm that produces the unit of good  $T$  ignore taxes. The consumer is better off after the trade and the firm is no worse off since its marginal cost is not more than  $P_\beta$ .

## 8.5 Additional Taxes

Taxing good  $T$  has resulted in an inefficient equilibrium. A potential solution is to tax both goods at the same rate  $t$ . That is, to set

$$P_T^C = P_T^F(1 + t) \quad (8.6)$$

and

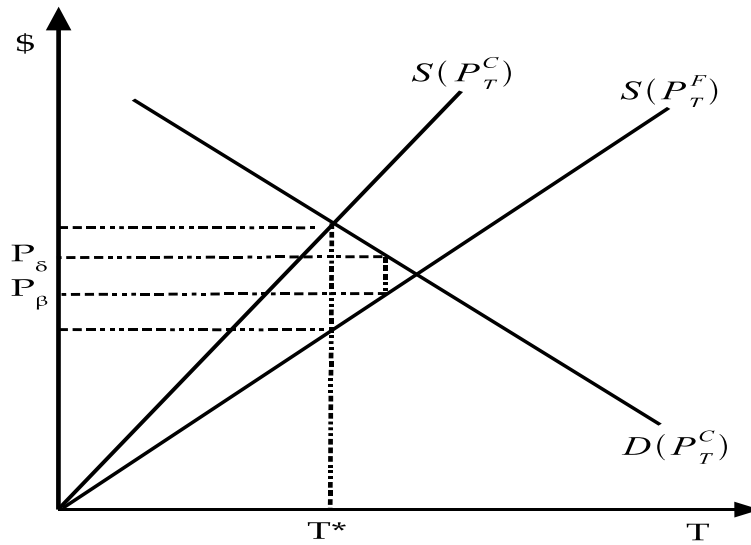


Figure 8.5: A Pareto-Improving Trade

$$P_U^C = P_U^F(1+t) \quad (8.7)$$

The efficiency criterion that was violated when only good  $T$  was taxed is now satisfied ( $ABC$  in the equation below stands for aggregate budget constraint).

$$\text{Slope of } ABC = -\frac{P_U^C}{P_T^C} = -\frac{P_U^F(1+t)}{P_T^F(1+t)} = -\frac{P_U^F}{P_T^F} = -\frac{MC_U}{MC_T} = \text{Slope of PPF} \quad (8.8)$$

The two slopes are equal, suggesting that the new equilibrium is efficient. Unfortunately, this is not true. The problem is that consumers also choose between consumption (goods  $T$  and  $U$ ) and leisure. When all goods except leisure are taxed, the slope of the PPF is

$$\frac{\Delta X}{\Delta l} = -\frac{MC_l}{MC_X} = -\frac{w}{P_X^F} \quad (8.9)$$

where  $X$  represents aggregate consumption. The slope of the aggregate budget constraint is

$$-\frac{w}{P_X^C} \quad (8.10)$$

Since the consumption good  $X$  is taxed,  $P_X^C > P_X^F$  so that

$$\text{Slope of ABC} = -\frac{w}{P_X^C} > -\frac{w}{P_X^F} = -\frac{MC_l}{MC_X} = \text{Slope of PPF} \quad (8.11)$$

Thus, the equilibrium will not be efficient. Taxing both goods does not resolve the inefficiency problem. If the government could tax leisure, it could resolve the problem by taxing consumption and leisure equally. But no one knows how to tax leisure.

## 8.6 Problems

1. Suppose that consumers choose between two goods, consumption (good  $X$ ) and leisure (good  $l$ ). The government decides to levy a tax on wages to pay for public goods. The wage that firms must pay  $w^F$  is equal to  $1 + t$  multiplied by the wage that consumers receive  $w^C$ . That is,

$$w^F = (1 + t)w^C \quad (8.12)$$

The government does not tax good  $X$ .

- (a) Using a diagram, construct the supply of labor for  $w^F$  (i.e., construct  $S(W^F)$ ). (Remember that in this setting, firms respond to  $w^F$  and consumers respond to  $w^C$ .)
- (b) Show the equilibrium values for  $L$ ,  $w^C$ , and  $w^F$  on your diagram and show the government's total tax revenue.
- (c) Would the equilibrium value for  $L$  be higher or lower if the government removed the tax?
- (d) Prove that your equilibrium is inefficient and then show it graphically by drawing the PPF and the aggregate budget constraint.

## 8.7 Solutions

1. (a) Figure 8.6 shows how to derive  $S(w^F)$  from  $S(w^C)$ . The first step is to pick a consumer wage rate such as  $w_1^C$ . At this price the height of the supply curve  $S(w^C)$  is the total amount of labor supplied,  $L_1$ . If the consumer receives  $w_1^C$  the firm must be paying a wage rate of  $w_1^F = w_1^C + tw_1^C$ . The height of the supply curve  $S(w^F)$  is therefore the height of the supply curve  $S(w^C)$  plus  $tw_1^C$ . In general, the height of the supply curve  $S(w^F)$  is the height of the supply curve  $S(w^C)$  plus  $tw^C$ . This general relationship allows me to plot the supply curve  $S(w^F)$  using the supply curve  $S(w^C)$ .

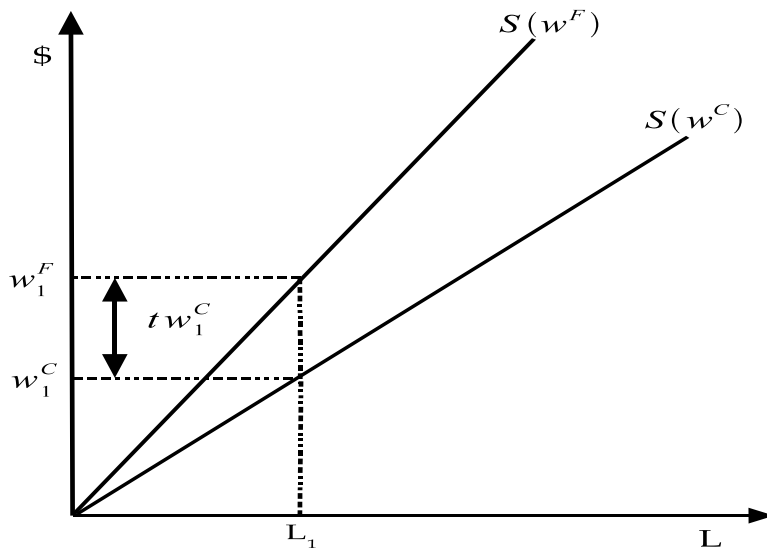


Figure 8.6:

- (b) Figure 8.7 shows the equilibrium values for  $L^*$ ,  $w^{C*}$ , and  $w^{F*}$ . The government's total tax revenue is the box  $w^{C*}$ - $w^{F*}$ - $A$ - $B$ .
- (c) If wages were not taxed, the supply curve  $S(w^F)$  would be equal to  $S(w^C)$  and the equilibrium point would be the intersection of  $S(w^C)$  and  $D(w^F)$ . Figure 8.7 shows this intersection and the resulting equilibrium wage rate  $w^{NT}$  and employment  $L^{NT}$ .  $L^{NT}$  is larger than  $L^*$ , meaning that employment would be higher without the tax.

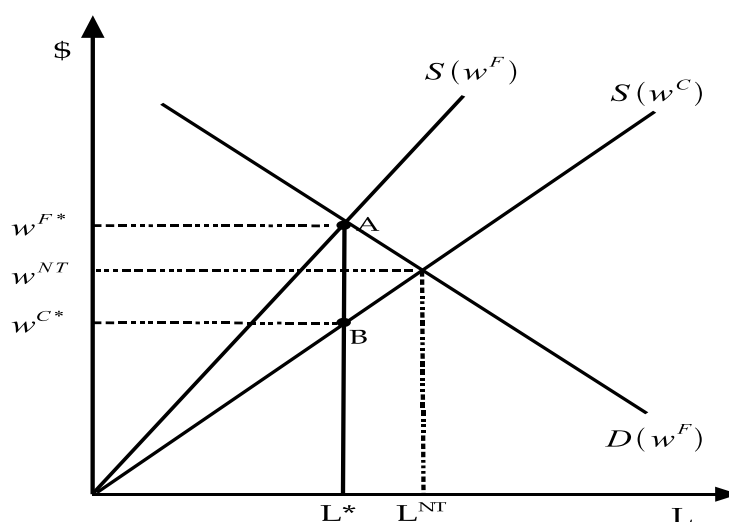


Figure 8.7:

- (d) If all consumers face the same prices, the slope of every consumer's budget constraint is

$$-\frac{w^C}{P_X^C} \quad (8.13)$$

This is therefore also equal to the slope of the consumers' indifference curves at their points of consumption. The slope of the PPF is equal to

$$\frac{\Delta X}{\Delta l} = -\frac{MC_l}{MC_X} = -\frac{w^F}{P_X^F} = -\frac{w^C(1+t)}{P_X^F} \quad (8.14)$$

The denominators in the two equations above are equal because  $P_X^C = P_X^F$  (recall that good  $X$  is untaxed by assumption), but the numerators are not equal because  $w^F = w^C(1+t) > w^C$ . The equilibrium is therefore inefficient. Figure 8.8 depicts the situation.

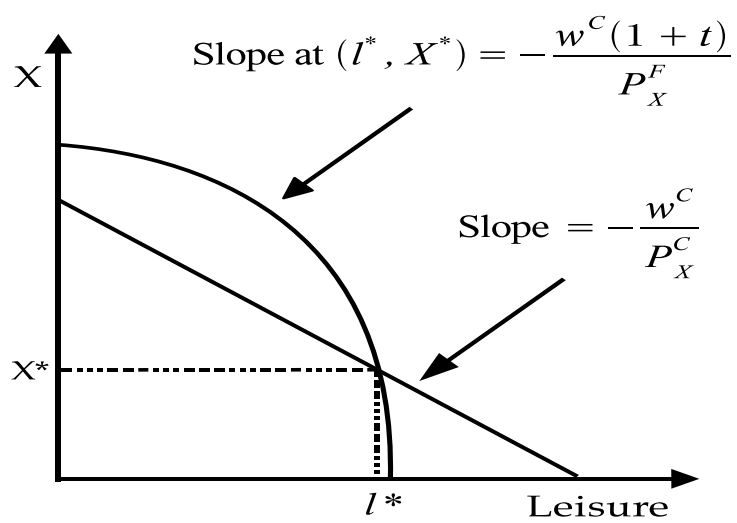


Figure 8.8: