Chapter 6

The Present and The Future

6.1 Calculating Present Values

I showed in chapter 4 that firms hire labor until the additional cost of hiring more labor equals the revenue that labor will earn. Now I want to determine how firms choose their level of capital. The problem is that capital is durable; if a firm pays for capital today, it doesn't have to pay for it again tomorrow (I shall ignore maintenance costs for the moment). Table 6.1 illustrates the problem for a farmer deciding whether to buy a tractor that will last for one time period.

Time Period	**Cost**	**Revenue**
0	900	0
1	0	1000

Table 6.1: The Cost and Benefit of a Tractor

The farmer pays $900 (the price of the tractor) today for her tractor. As a result, she receives $1,000 in additional revenue next period. Before she can compare revenue with cost, she must convert her revenue of $1,000 into today's dollars. This is known as finding the present value.

Present values (PV) are calculated using the interest rate. The interest rate, denoted r, allows a consumer or firm to calculate how much money the bank will give him next period if he lets it hold his money until then. Suppose for simplicity that time periods are days and that interest is paid daily. If a consumer or firm puts $10 in the bank today (time 0), tomorrow (time 1) the bank will give him:

$$X_1 = 10 + 10r = (1 + r)10 \tag{6.1}$$

The bank gives him back his deposit plus the product of the deposit and the interest rate. The farmer in the problem above though wants to translate next period's dollars into today's dollars. I can do this by manipulating the formula above. Suppose that I offer to give you \$10 tomorrow if you loan me \$$X$ today. How much will you be willing to loan me? To answer this question, I ask what \$$X_0$ today will become tomorrow. Using equation 6.1, I get

$$10 = (1 + r)X_0 \Rightarrow X_0 = \frac{10}{1 + r} \tag{6.2}$$

The farmer can use equation 6.2 to solve her tractor problem by converting \$1,000 in tomorrow's dollars into today's dollars. If R_1 is revenue earned tomorrow, she calculates its present value—its value in today's dollars—as

$$PV(R_1) = \frac{1000}{1 + r} \tag{6.3}$$

So the value of \$1,000 next period in this period's dollars depends upon the interest rate. The farmer will want to buy the tractor if the present value of the additional revenue due to the tractor is larger than the present value of the tractor's cost. She should buy it if

$$900 \leq PV(1000) \Rightarrow 900 \leq \frac{1000}{1 + r} \Rightarrow (1 + r)900 \leq 1000 \Rightarrow r \leq \frac{1}{9} \tag{6.4}$$

Now suppose that the farmer's problem is as follows:

Time Period	**Cost**	**Revenue**
0	900	0
1	0	500
2	0	500

Table 6.2: The Cost and Benefit of a Tractor

If she pays \$900 today for her tractor she receives \$500 in additional revenue tomorrow and \$500 in additional revenue the day after tomorrow.

She finds the present value of R_1 using equation 6.3. To find the present value of R_2, our farmer first converts it into tomorrow's dollars by multiplying by $1/(1+r)$. Then she converts it from tomorrow's dollars into today's dollars by again multiplying by $1/(1+r)$.

$$PV(R_2) = \frac{500}{1+r}\frac{1}{1+r} = \frac{500}{(1+r)^2} \tag{6.5}$$

You may now be able to guess the general formula for calculating the present value of dollars earned at time t from equations 6.3 and 6.4. It is

$$PV(X_t) = \frac{X_t}{(1+r)^t} \tag{6.6}$$

Returning to the farmer and her tractor, the present value of her additional revenue is

$$PV(R) = \frac{500}{(1+r)} + \frac{500}{(1+r)^2} \tag{6.7}$$

while the present value of her cost is \$900. She should purchase the tractor if $PVR \geq PVC$.

The farmer can use equation 6.6 to solve more complicated problems than the two simple examples above. To do so, she uses equation 6.6 to calculate the present value of all costs and revenues related to her tractor. I shall call the present value of her total costs PVC and the present value of her total revenues PVR. She buys the tractor if

$$PVR \geq PVC \tag{6.8}$$

She calculates her PVC as

$$PVC = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n} = \sum_{t=0}^{n} \frac{C_t}{(1+r)^t} \tag{6.9}$$

The variable n in equation 6.9 represents the number of time periods since the tractor was purchased. C_t represents the cost in time t (C_0 is the purchase price, C_1 is total maintenance costs one period after the purchase, etc.). The PVR is calculated analogously.

$$PVC = R_0 + \frac{R_1}{1+r} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} = \sum_{t=0}^{n} \frac{R_t}{(1+r)^t} \tag{6.10}$$

Table 6.3 illustrates these calculations. Assume that $r = 0.1$ and that the tractor stops working 3 periods after it was purchased (i.e., at the end of period 3).

Time Period	**Cost**	**PVC**	**Revenue**	**PVR**
0	2050	2050	0	0
1	100	90.91	1000	909.09
2	100	82.64	900	743.8
3	100	75.13	800	601.05
Total	2350	2298.68	2700	2253.94

Table 6.3: The Cost and Benefit of a Tractor

This example shows that our farmer should not buy the tractor since $PVC > PVR$. Note that if she had just compared cost and revenue, she would have purchased the tractor.

6.2 Assets That Pay Forever

What is the present value of an asset A (e.g., a piece of land) that pays $\$X$ per period forever? If the asset pays nothing at time zero, the present value of the asset is

$$PV(A) = \frac{X}{(1+r)^1} + \frac{X}{(1+r)^2} + \frac{X}{(1+r)^3} + \ldots \tag{6.11}$$

where the three dots indicate that the terms continue forever. The maximum that one would be willing to pay, denoted P, is

$$P = \frac{X}{(1+r)^1} + \frac{X}{(1+r)^2} + \frac{X}{(1+r)^3} + \ldots \tag{6.12}$$

Multiplying both sides by $1 + r$ gives

$$P(1+r) = X + \frac{X}{(1+r)^1} + \frac{X}{(1+r)^2} + \frac{X}{(1+r)^3} + \ldots \tag{6.13}$$

If I subtract X from both sides I have

$$P(1+r) - X = \frac{X}{(1+r)^1} + \frac{X}{(1+r)^2} + \frac{X}{(1+r)^3} + \dots \tag{6.14}$$

But the right-hand side is now equal to P so I can write

$$P(1+r) - X = P \Rightarrow rP - X = 0 \Rightarrow rP = X \Rightarrow P = \frac{X}{r} \tag{6.15}$$

The value of the asset is X divided by the interest rate. If, for example, X equals \$100 and r equals 0.1, the value of the asset is \$1,000.

6.3 Supply and Demand for Credit

The interest rate can be interpreted as the price of credit. When I deposit money in the bank, I am giving the bank credit until I withdraw it. In return for the credit, the bank pays me interest. I can analyze the supply and demand for credit using the same tools I've used in previous chapters. To do so I will consider decisions involving only two time periods: this simplifies the calculations and helps to bring out the intuition.

Credit is demanded by firms who wish to invest in new capital. I can determine how much credit they will demand by calculating the rate of return for potential investments. Suppose that a firm pays \$900 for a piece of capital at time 0 and earns an additional \$1,000 at time 1. The rate of return, denoted ρ, on this investment is

$$\rho = \frac{1000 - 900}{900} = \frac{1}{9} \tag{6.16}$$

The rate of return tells the firm much additional money each dollar invested in new capital earns it. A firm can also choose to put its money in the bank, in which cases its rate of return is the interest rate. A firm will therefore demand credit for an investment project if the rate of return of the project exceeds the interest rate. It will not demand credit if the rate of return of a project is below the interest rate. The demand for credit will therefore decrease when the interest rate rises and increase when the interest rate falls.

Credit is supplied by consumers, not banks. Consumers deposit their savings in bank accounts and banks then lend this money to firms. The banks don't determine the supply, they act as intermediaries.

Now consider a consumer who lives two time periods. She earns income Y_0 in time period 0 and income Y_1 in time period 1. She consumes C_0 in time period 0 and C_1 in time period 1 (assume for simplicity that P_C is equal to 1 in both periods). She is free to borrow or lend so long as she repays her loans. I shall assume that she spends all of her money before her death. In other words, the present value of her consumption must equal the present value of her income. That is,

$$PV(C) = C_0 + \frac{1}{1+r}C_1 = Y_0 + \frac{1}{1+r}Y_1 = PV(Y) \qquad (6.17)$$

I can use equation 6.17 to find the consumer's lifetime budget constraint: C_0 and C_1 are the two goods and Y_0 and Y_1 are her income. Solving equation 6.17 for C_0 gives

$$C_0 = Y_0 + \frac{1}{1+r}Y_1 - \frac{1}{1+r}C_1 \qquad (6.18)$$

The slope of the budget constraint is $-1/(1+r)$. The Y-axis intercept is $C_0 = Y_0 + 1/(1+r)Y_1$ and the X-axis intercept is $C_1 = (1+r)Y_0 + Y_1$. I can graph the consumer's budget constraint using the information in equation 6.18 and the fact that the consumer can always choose to set C_0 equal to Y_0 and C_1 equal to Y_1 (i.e., neither consume nor borrow). Figure 6.1 illustrates.

When $Y_0 - C_0$ is positive, the consumer is saving money; when $Y_0 - C_0$ is negative, she is borrowing. A change in the interest rate changes the slope of the budget constraint. If the interest rate rises, the budget constraint becomes flatter, as shown in figure 6.2 (note that setting C_0 equal to Y_0 and C_1 equal to Y_1 is still possible and hence on the budget constraint). Figure 6.2 illustrates the rise in the interest rate and my consumer's possible reactions.

I can't be sure how an increase in the interest rate will affect savings. The consumer can locate anywhere between points A and D. Points between A and B represent a decrease in savings while points between B and D represent an increase in savings. I would expect my consumer to increase her saving, but this need not be the case. Suppose for example that she is saving with a target in mind (e.g., college tuition); if the interest rate rises, she can save less and still meet her target.

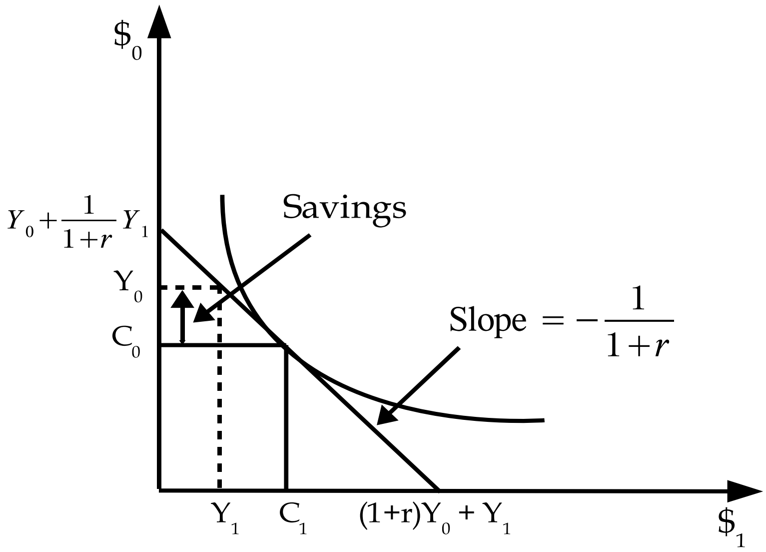

Figure 6.1: The Consumer's Choice Problem

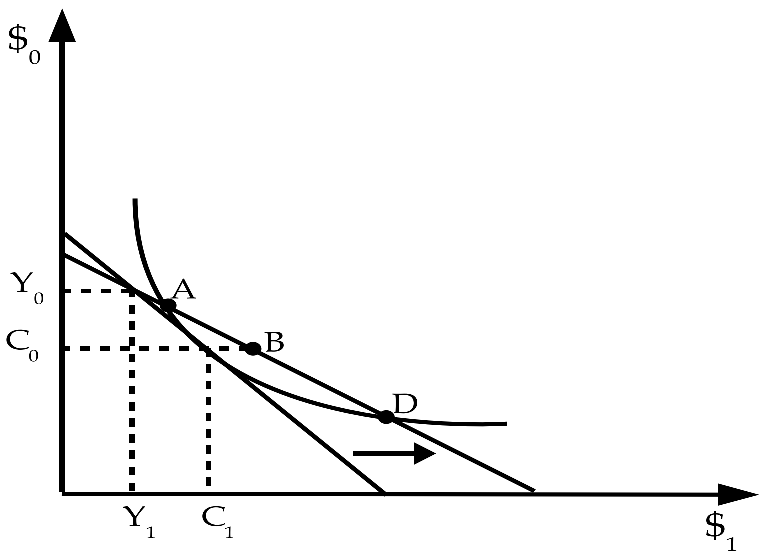

Figure 6.2: A Rise in the Interest Rate

Many economists believe that a rise in the interest rate increases savings, but not by very much. If true, this would mean that the supply curve for credit would be a nearly vertical line. Figure 6.3 shows a possible equilibrium in the market for credit.

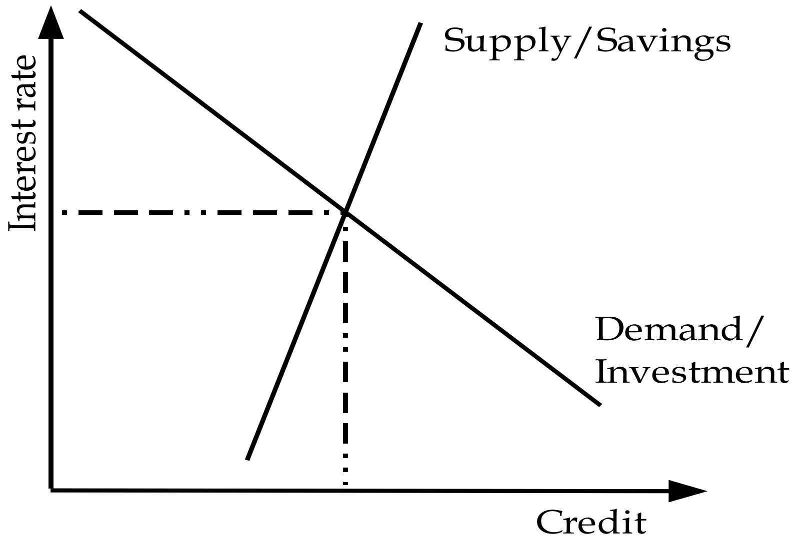

Figure 6.3: The Supply and Demand of Credit

6.4 Efficiency

Is this section I will show that the equilibrium that I derived in the previous section is efficient if all consumers face the same interest rate. If this assumption is true, the slope of the budget constraint is $-1/(1+r)$ for all consumers. Their indifference curves therefore have the same slope at their points of consumption. As a result, they can't trade amongst themselves and make someone better off without making someone else worse off.

I assume also that the economy produces two goods, C_0 and C_1: consumption for today and consumption for tomorrow, respectively. The slope of the PPF shown in figure 6.4 is equal to the ratio of the marginal costs of production in the two periods. Tomorrow's marginal cost must be converted into today's dollars by dividing it by $(1+r)$. I shall also assume that both industries are perfectly competitive. In equilibrium therefore, marginal cost is equal to price. Thus,

$$\frac{\Delta C_0}{\Delta C_1} = -\frac{\frac{MC_1}{1+r}}{MC_0} = -\frac{\frac{P_1}{1+r}}{P_0} = -\frac{P_1}{P_0(1+r)} = -\frac{1}{1+r} \tag{6.19}$$

Another way to derive the slope of the PPF is to note that a firm can give up one unit of production today and instead use it for an investment project with a rate of return of $1+\rho$. Next period the firm can sell its $1+\rho$ additional units of output. The PPF will be bowed outwards because the firm will want to do investment projects with a high rate of return—and therefore a relatively flat slope—first. As it does more and more projects, the rate of return of each additional project falls, and the slope of the PPF becomes steeper. The slope of the PPF at any given point is the rate at which it can convert production today into production tomorrow (i.e., $-\frac{1}{1+\rho}$). In equilibrium that rate of return of the last investment undertaken by the firm is equal to the interest rate. Thus, in equilibrium, equation 6.19 must be true.

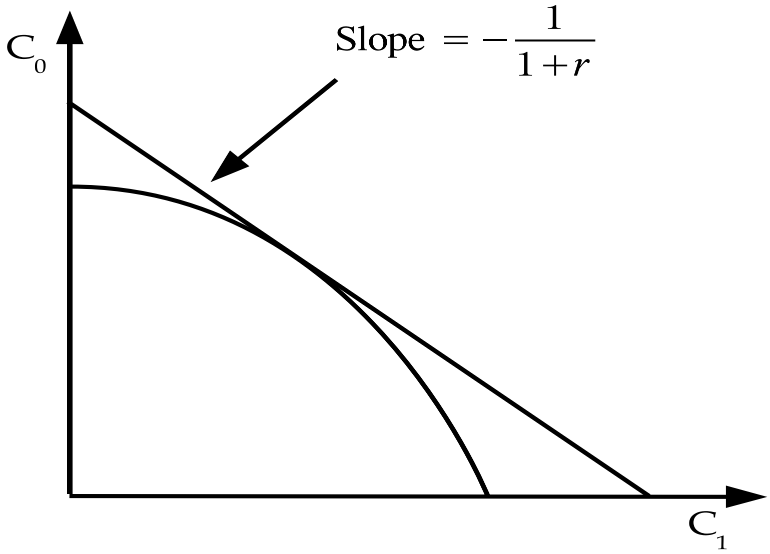

Figure 6.4: Efficiency in the Market for Credit

Figure 6.4 combines the PPF and the aggregate budget constraint. The slope of the PPF at the point of production and the slope of every consumer's indifference curve at the point of consumption are equal so the equilibrium is efficient.

6.5 Problems

1. Suppose that a farmer deciding whether or not to buy a tractor is faced with the following information (where revenue represents additional revenue).

Time Period	**Cost**	**Revenue**
0	900	500
1	150	575

Table 6.4: The Cost and Benefit of the Tractor

   (a) Calculate the present value of the cost and the additional revenue if the interest rate r is equal to 0.1. Should the farmer purchase the tractor?

   (b) What value of r will make the farmer indifferent between purchasing the tractor and not purchasing the tractor? If r is smaller than this value, should the farmer purchase the tractor?

2. Suppose that a wheat farmer is trying to decide whether or not to buy a tractor. The tractor costs $1,000. It will earn $600 in additional revenue in the year that it is purchased and $500 in additional revenue in the following year; then it breaks.

Time Period	**Cost**	**Revenue**
0	1000	600
1	0	500

Table 6.5: The Cost and Benefit of a Tractor

   (a) If r is the annual interest rate, what is the present value of the additional revenue earned by tractor?

   (b) Let r equal 1/9. Should the farmer purchase the tractor? Explain.

(c) What is the largest r at which the farmer should purchase the tractor?

(d) Now suppose that the farmer is offered a special deal by the John Deere dealer: payment is delayed by a year. The farmer pays $0 in year 0 and $1000 in year 1; the additional revenue is unchanged. Should the farmer buy the tractor if r equals 1/9?

(e) Finally, the farmer can also install an irrigation system that costs $X. The system takes one year to install, but beginning in year 1 it will yield an additional $100 of wheat and continue to do so every year for the rest of time. For what values of r should the farmer install the irrigation system?

3. A firm has 3 investment projects that it is considering.

Project	**Investment Today**	**Return Tomorrow**
1	900	950
2	775	850
2	1975	2100

Table 6.6: The Cost and Benefit of a Tractor

(a) Calculate the rate of return for each project and rank the projects from best to worst.

(b) Suppose that the interest rate r equals 0.075. Which projects should the firm undertake?

4. Consider a consumer who lives for two periods. She earns Y_0 in time period 0 and Y_1 in time period 1. Use C_0 and C_1 to denote her consumption in time periods 0 and 1 respectively. The consumer is allowed to borrow or lend in time period 0. If she borrows, she must repay her debt in time period 1. She must spend all of her income before dying.

If the interest rate is r, the above conditions can be written as:

$$(Y_0 - C_0) + \frac{1}{1+r}(Y_1 - C_1) = 0 \tag{6.20}$$

(a) Rearrange equation 6.20 to get a budget constraint for the consumer treating C_0 and C_1 as goods. (Hint: solve equation 6.20 for C_0.) Draw the resulting budget constraint.

(b) Now suppose that the interest rate increases to ${}'r$. Show the effect of this change on the consumer's budget constraint. Explain intuitively why an increase in the interest rate has this effect on a consumer's budget constraint.

(c) How will the increase in the interest rate affect the consumer's choice of C_0 and C_1? Draw a diagram to support your argument.

5. You graduate and are offered a job with a salary of $50,000 dollars this year and $60,000 next year. The interest rate is 5 percent (i.e., r equals 0.05).

   (a) Draw your budget constraint. What is the slope of the constraint? Explain.

   (b) Suppose that you save $10,000 this year. Draw a relevant indifference curve and indicate the amount that you save. Show also the amount of principal plus interest that you get next year.

   (c) Suppose that the interest rate is raised to 0.07. Will your savings increase or decrease (use your diagram in part 5b and take the answer that it gives you)?

   (d) Draw a supply curve of savings that is consistent with your answer in part 5c.

6.6 Solutions

1. (a) Since $PVC > PVR$ (see below), the farmer shouldn't purchase the tractor.

$$PV(C) = 900 + \frac{150}{1+r} = 900 + \frac{150}{1.1} = 1036.4 \qquad (6.21)$$

$$PV(R) = 500 + \frac{575}{1+r} = 500 + \frac{575}{1.1} = 1022.7 \qquad (6.22)$$

(b) I can solve this problem by finding the value of r at which $PVC = PVR$. If the interest rate is smaller than 0.0625, the farmer should purchase the tractor.

$$900 + \frac{150}{1+r} = 500 + \frac{575}{1+r} \Rightarrow \frac{425}{1+r} = 400 \Rightarrow$$
$$400(1+r) = 425 \Rightarrow r = 0.0625 \tag{6.23}$$

2. (a) The present value of the additional revenue is

$$PVR = 600 + \frac{500}{1+r} \tag{6.24}$$

(b) Yes. The farmer should purchase the tractor because $PVR > PVC$.

$$PVR = 600 + \frac{500}{1+\frac{1}{9}} = 600 + \frac{500}{10/9} = 600 + 450 = 1050 >$$
$$1000 = PVC \tag{6.25}$$

(c) The farmer is indifferent if $PVR = PVC$. That is, if

$$1000 = 600 + \frac{500}{1+r} \Rightarrow \frac{500}{1+r} = 400 \Rightarrow 400(1+r) = 500$$
$$\Rightarrow r = \frac{1}{4} \tag{6.26}$$

(d) Yes. The tractor is a good investment since

$$600 + \frac{500}{1.09} \geq \frac{1000}{1.09} \tag{6.27}$$

(e) Using the formula derived in this chapter, the investment is worth $\$100/r$. The farmer should install the system whenever

$$\frac{100}{r} \geq X \Rightarrow r \leq \frac{100}{X} \tag{6.28}$$

3. (a) The projects' rates of return are as shown below. They rank as follows: $ROR2 > ROR3 > ROR1$.

$$ROR_1 = \frac{950 - 900}{900} = 0.055 \tag{6.29}$$

$$ROR_2 = \frac{850 - 775}{775} = 0.097 \tag{6.30}$$

$$ROR_3 = \frac{2100 - 1975}{1975} = 0.063 \tag{6.31}$$

   (b) The firm should undertake any investment project with a rate of return greater than the interest rate. If this is true, then the money invested in the project makes more than if it were left in the bank. Only project 2 has a rate of return larger than the interest rate, making it the only project that the firm should undertake.

4. (a) Solving the equation for C0 gives

$$C_0 = Y_0 + \frac{1}{1+r}Y_1 - \frac{1}{1+r}C_1 \tag{6.32}$$

   The slope of the budget constraint is $-1/(1 + r)$. The intercepts are $C_0 = Y_0 + 1/(1 + r)Y_1$ and $C_1 = (1 + r)Y_0 + Y_1$. Figure 6.5 illustrates. When $Y_0 - C_0$ is positive, the consumer is saving; when $Y_0 - C_0$ is negative, she is borrowing.

   (b) A change in the interest rate changes the slope of the budget constraint. If the interest rate rises, the budget constraint becomes flatter, as shown in figure 6.6. This is because savings earn more interest and so the consumer can consume more in time period 1 if she saves today. Conversely, borrowing is now more expensive, so she will owe more in time period 1 if she borrows, meaning that she can consume less in time period 0. Notice that the point (Y_1, Y_0) is on both budget constraints—the consumer can always consume her income and neither borrow nor save.

   (c) I can't be sure how an increase in the interest rate will affect C_0 and C_1. All I can say for sure is that the consumer will locate somewhere between points A and D on figure 6.7.

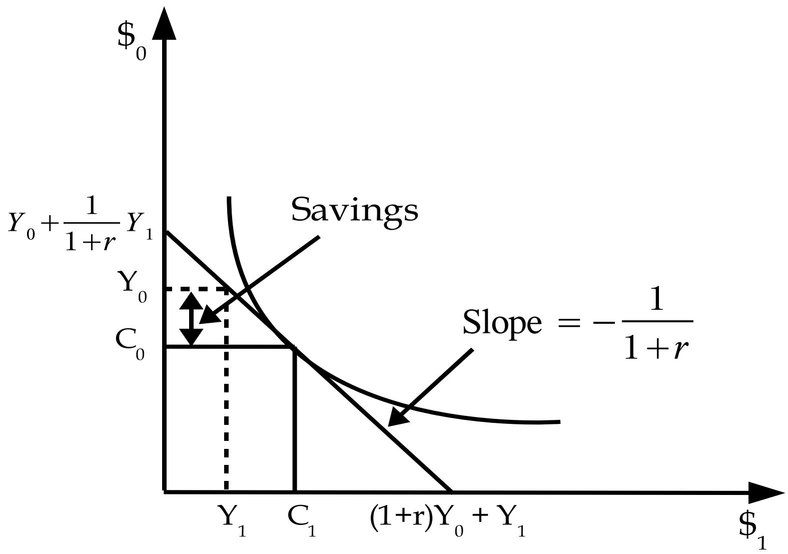

Figure 6.5: The Consumer's Choice

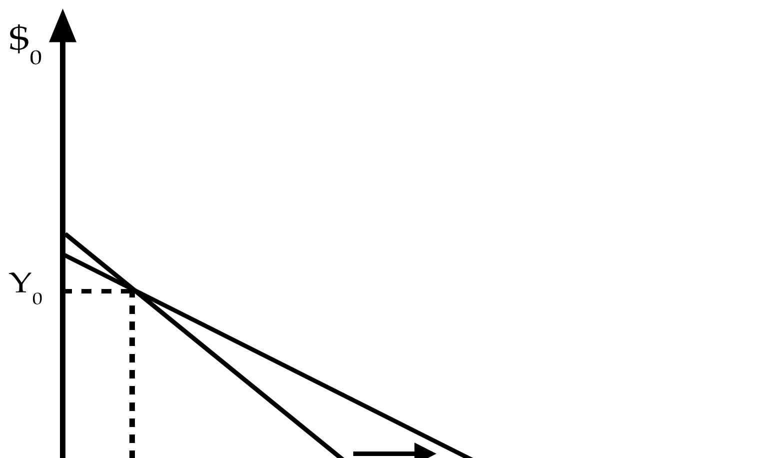

Figure 6.6: A Rise in the Interest Rate

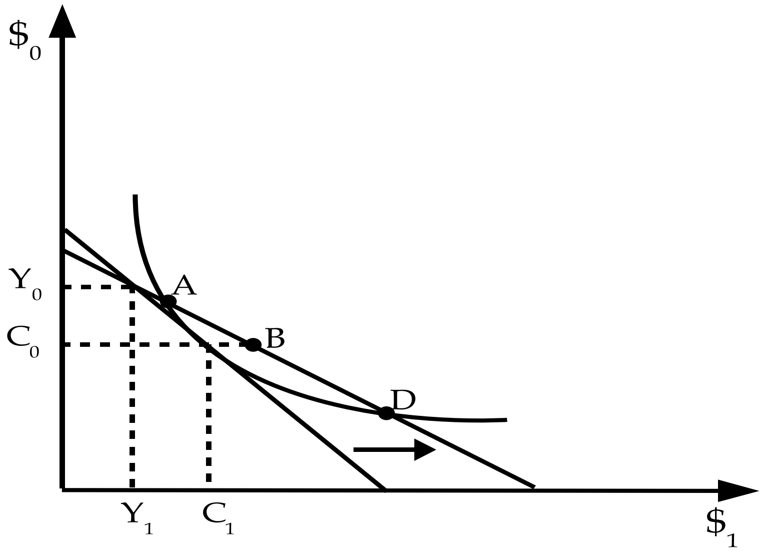

Figure 6.7: The Consumer's New Choice

5. (a) Recall that the budget constraint can be written as

$$C_0 = Y_0 + \frac{1}{1+r}Y_1 - \frac{1}{1+r}C_1 \tag{6.33}$$

Let C_1 equal zero and

$$C_0 = 50000 + \frac{1}{1+0.05}60000 \approx 107142.8 \tag{6.34}$$

Let C_0 equal 0 and

$$0 = 50000 + \frac{1}{1+0.05}60000 - \frac{1}{1+0.05}C_1 \Rightarrow C_1 = 112500 \tag{6.35}$$

The slope is

$$-\frac{1}{1+r} = -\frac{1}{1.05} \approx -0.95 \tag{6.36}$$

Putting the intercepts, slopes, and the fact that it is always possible to have C_0 equal 50,000 and C_1 equal 60,000 together graphically yields figure 6.8.

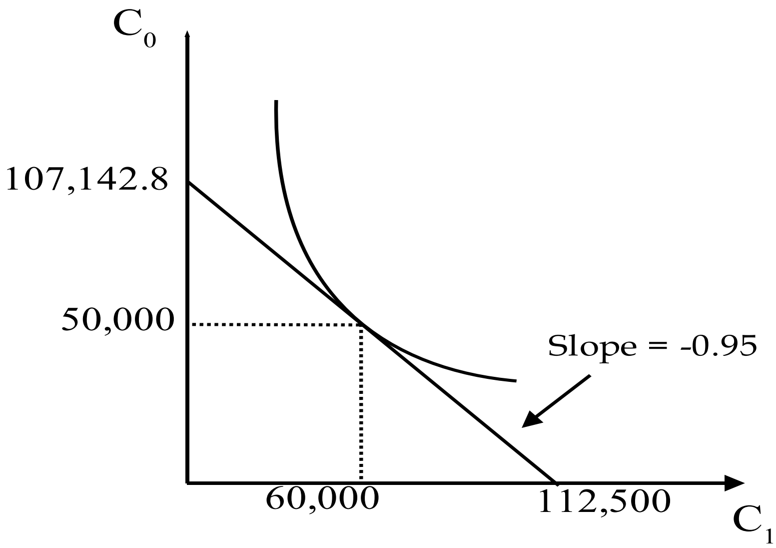

Figure 6.8: The Consumer's New Choice

The slope of the budget constraint represents your ability to convert today's consumption into tomorrow's consumption. If you increase tomorrow's consumption by one dollar, you must decrease today's consumption by \$0.95 (i.e., you save \$0.95 today and tomorrow the bank gives you one dollar). If you decrease tomorrow's consumption by one dollar, you gain \$0.95 today (i.e., you borrow \$0.95 today and tomorrow you must pay the bank one dollar).

(b) Figure 6.9 answers the question. I haven't drawn interest separately from principal. To do so, mark a spot just to the left of C_1 as 70,000. The distance between this point and C_1 is interest.

(c) My figure 6.10 shows no change in savings, but your answer depends upon the graph that you drew: it is possible for savings to increase, decrease, or remain constant.

(d) The supply curve shown below is consistent with my answer to part 5c, which showed no change in savings when the interest rate changed (i.e., savings do not depend upon the interest rate).

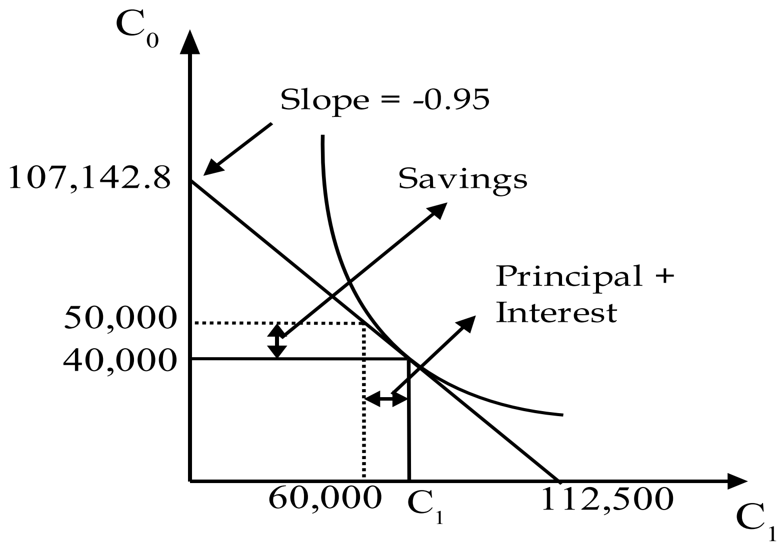

Figure 6.9: The Consumer's Saving

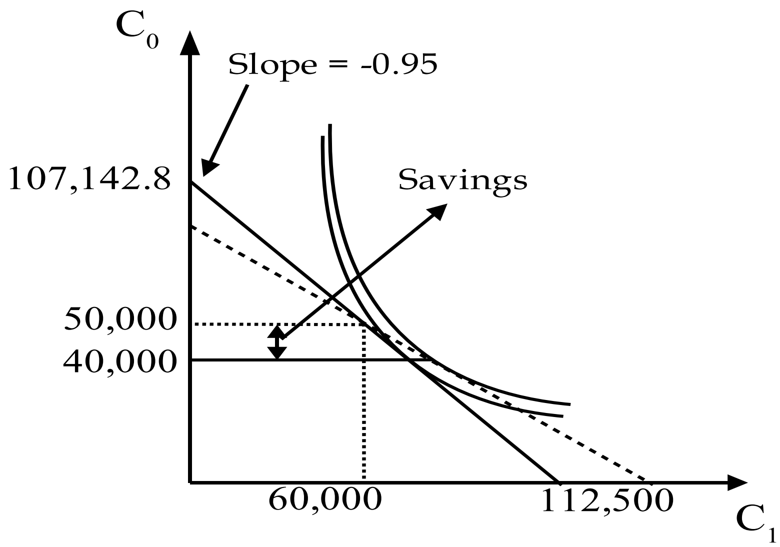

Figure 6.10: An Increase in the Interest Rate

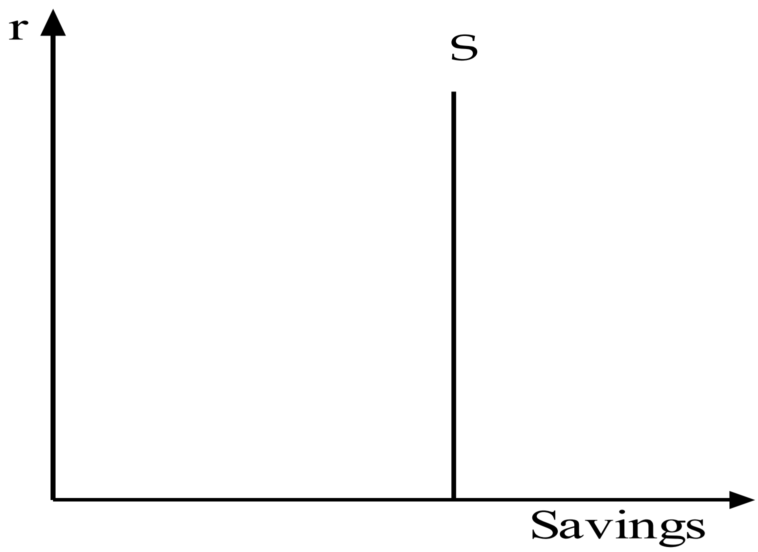

Figure 6.11: The Supply of Credit