

# Chapter 4

## The Theory of a Single Firm

I shall consider in this chapter a small firm that uses capital  $K$  and labor  $L$  to produce an output good  $X$ . I assume throughout the chapter that the firm is in a perfectly competitive industry as defined below.

**Definition 4** *An industry is perfectly competitive if there are so many small firms that no firm can affect the price (firms are price-takers) and firms can enter and exit without cost.*

### 4.1 The Long Run

In the long run, a firm can change both capital and labor. I shall assume that constant returns to scale hold: a proportional change in inputs results in a equal proportional change in output. For example, doubling inputs will double output; halving inputs will halve output.

The curves in figure 4.1 are called iso-quants. They show all the combinations of capital and labor that will produce a given level of output. The iso-quant labeled  $X = 1$ , for example, shows all the combinations of capital and labor that produce one unit of output.

In the long-run, firms can vary their capital and their labor. In the short-run though, I shall assume that the firm can vary its labor inputs but not its capital inputs (i.e., its capital stock is fixed). I make this assumption because capital investment is usually time-intensive—building a factory can take years—while additional labor can be hired quickly.

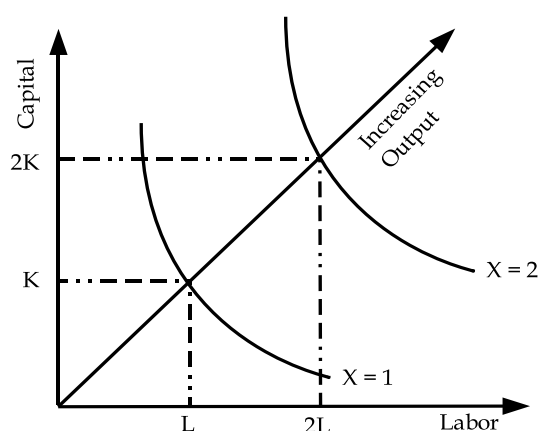


Figure 4.1: Constant Returns to Scale

## 4.2 Types of Cost

The total amount that a firm spends on its fixed input (capital) is called total fixed cost. The total amount that a firm spends on its variable input (labor) is called total variable cost. Total cost ( $TC$ ) for a given level of output is the total amount that a firm spends on inputs. It is equal to the sum of total fixed cost ( $TFC$ ) and total variable cost ( $TVC$ ).

$$TC = TFC + TVC \quad (4.1)$$

The average cost ( $AC$ ) for a given level of output is the amount that the firm spends on inputs per unit of output. It is equal to total cost divided by total output.

$$AC = \frac{TC}{X} \quad (4.2)$$

A firm that spends \$500 to make 10 units of  $X$  would have an average cost of \$50. This does not mean that each  $X$  cost \$50—it means only that the *average* cost per unit is \$50.

Marginal cost ( $MC$ ) at a given level of output is the change in total cost that a firm incurs if it changes output.

$$MC = \frac{\Delta TC}{\Delta X} \quad (4.3)$$

### 4.3 Marginal Cost

To understand marginal cost, one must understand—and accept—the law of diminishing returns.

**Definition 5** The law of diminishing returns *states that if one input is fixed, after some point, additional equal increments of the variable input will produce ever decreasing increments of output.*

Figure 4.2 develops the concept of marginal cost. The left-hand pane illustrates the law of diminishing returns: as the firm increases its labor inputs, it gets less additional output per unit of labor. In figure 4.2, the first increase of labor equal to  $\Delta L$  increases output by  $\Delta X_1$ . The second increase in labor equal to  $\Delta L$  increases output by  $\Delta X_2$ , which is much smaller than  $\Delta X_1$ .

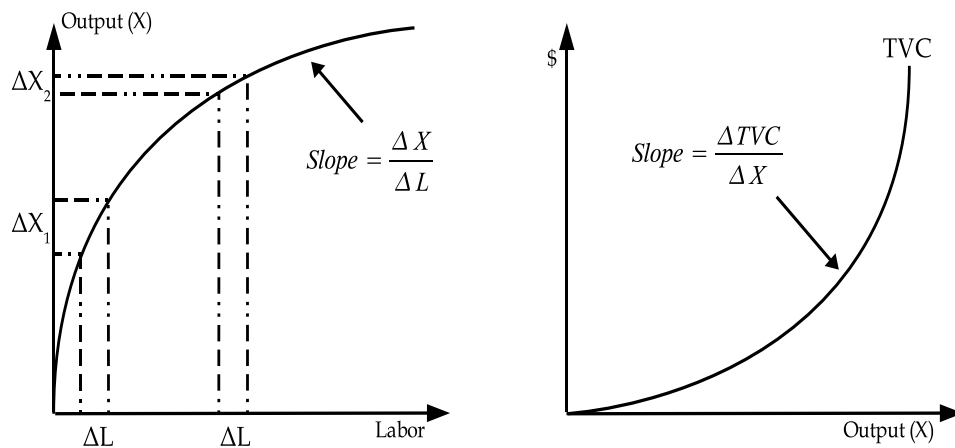


Figure 4.2: The Marginal Product of Labor and Total Variable Costs

The right-hand pane illustrates how the firm's total variable cost—the wage rate times  $L$ —varies with output. As the firm increases output, its total variable cost increases at an increasing rate, despite a constant wage rate  $w$ . Costs increase at this increasing rate because each unit of output requires more labor than the preceding unit of output (as we saw in the left-hand pane). Each worker receives the same wage, but the firm has to hire more workers per unit of output, increasing the firm's total variable cost at an increasing rate.

The slope of the TVC curve at a particular point is equal to the change in total variable cost over the change in output, which is marginal cost. Equation 4.4 rearranges the definition of marginal cost given in equation 4.3 to emphasize this more precise definition.

$$MC = \frac{\Delta TC}{\Delta X} = \frac{\Delta TFC}{\Delta X} + \frac{\Delta TVC}{\Delta X} = \frac{\Delta TVC}{\Delta X} \quad (4.4)$$

The last equality holds because changing output does not affect total fixed costs—by assumption, these costs are fixed. Thus,  $\Delta TFC/\Delta X$  is equal to zero and changing output only affects total variable costs. This means that the slope of the total variable cost curve is equal to marginal cost. Figure 4.3 shows a firm's marginal cost curve.

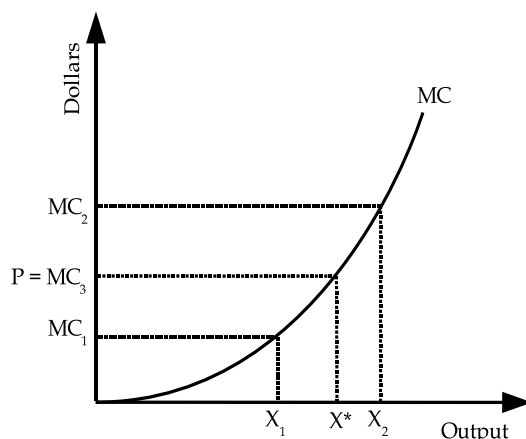


Figure 4.3: The Marginal Cost Curve

The optimal output for this firm is  $X^*$ . To see why, suppose that the firm produces  $X_1$ . The cost of making another unit of output,  $MC_1$ , is less than the revenue from that output,  $P$ , which means that expanding production will increase the firm's profit. Now suppose that the firm produces  $X_2$ . The cost of making one unit of output,  $MC_2$ , is greater than the revenue from that output  $P$ , which means that reducing production will increase the firm's profit. The optimal production point for the firm is therefore  $X^*$ . At  $X^*$ , the marginal cost ( $MC_3$ ) and benefit ( $P$ ) of changing output are equal—which means that changing output won't increase the firm's profit.

## 4.4 Total Cost

Marginal cost tells a firm its optimal level of output, but it doesn't tell it how much profit it will earn at that output level. To calculate its total profit, the firm must use total cost or average cost. Recall that total cost is the sum of total variable cost and total fixed cost. Figure 4.4 derives the total cost curve from the total variable cost and total fixed cost curves.

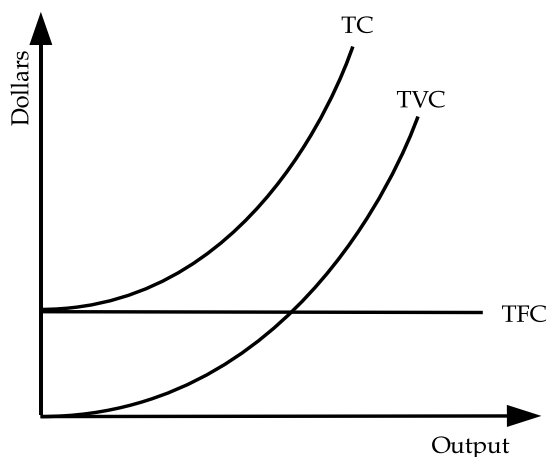


Figure 4.4: The Total Cost Curve

Notice that the slope of the total cost curve is the same as the slope of the total variable cost curve and that the vertical difference between the total cost curve and the total variable cost curve at any output level is equal to total fixed cost.

## 4.5 Average Cost

The average cost curve can be derived from the total cost curve—figure 4.5 shows how. The slope of the line from the origin to the total cost curve is equal to average cost: the rise is total cost, the run is output. The slope is thus total cost over output, which is the definition of average cost.

If one draws a line from the origin to the total cost curve at a very low level of output, it will have a very steep slope, indicating that the average cost is very high. If one draws lines for increasing levels of output, one will

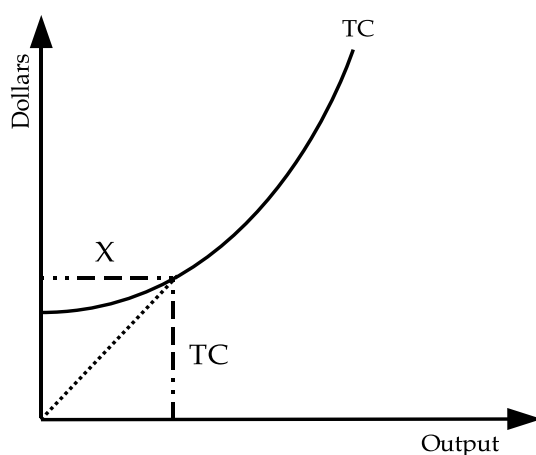


Figure 4.5: Calculating Average Cost

find that the slopes decrease for a while (average cost goes down) and then begin to increase again (average cost rises). To see why this is so, consider carefully equation 4.5.

$$AC = \frac{TC}{X} = \frac{TFC}{X} + \frac{TVC}{X} \quad (4.5)$$

Since total fixed cost does not change with output, the term  $TFC/X$  decrease as the firm's output increases. For example, suppose that total fixed cost is \$60 regardless of the firm's output level. When output is equal to one, the term  $TFC/X$  is equal to \$60. When output is equal to two, the term  $TFC/X$  is equal to \$30. When output is equal to three, the term  $TFC/X$  is equal to \$20. Notice that as output rises, the term  $TFC/X$  decreases at a decreasing rate.

Total variable cost, on the other hand, does change with output. In fact, I discovered earlier that it increases at an increasing rate when output rises. For example, suppose that a firm needs one unit of labor to produce one unit of output and 3 units of labor to produce two units of output (labor has diminishing returns). Suppose also that wages are \$1. The total variable cost of making one unit of output is \$1 (the firm's labor cost) so the term  $TVC/X$  also equals \$1. The total variable cost of making two units of output is \$3 (since the firm must hire three workers) so  $TVC/X$  is equal to \$1.5. Notice that the term  $TVC/X$  increases as output increases.

One can now see why average cost declines and then increases. Initially, increasing output decreases  $TFC/X$  by more than it increases  $TVC/X$  so average cost falls. Eventually however, increasing output increases  $TVC/X$  by more than it decreases  $TFC/X$  and average cost rises.

I showed earlier that firms use marginal cost to determine their optimal output level; why then do they care about average cost? As equation 4.6 shows, firms use average cost to calculate their profit.

$$\pi = TR - TC = P_X X - TC = P_X X - ACX \quad (4.6)$$

Why does total cost equal average cost times  $X$ ? Remember that average cost is defined as  $TC/X$ . Multiply  $TC/X$  by  $X$  and the result is  $TC$ .

Why can't firms use marginal cost to calculate total cost? For a given level of output, marginal cost tells a firm how much total costs will change if it changes output; it tells the firm nothing about its current total variable cost. It also tells the firm nothing about total fixed cost since they don't change when output changes.

## 4.6 The Complete Short-Run graph

Figure 4.6 depicts the marginal and average cost curves for a firm. It also shows the current market price  $P$  for the good that the firm produces. The firm can compute its optimal output level, total revenue, total cost, and profit using this figure.

The firm's ideal output level is  $X^*$ , the output level at which price equals marginal cost. The firm's total revenue is given by the box  $ABDE$  ( $P$  times  $X^*$ ). The firm's total cost is given by the box  $ABCF$  ( $AC$  times  $X^*$ ). The firm's profit is the box  $FCDE$  (total revenue minus total cost).

## 4.7 The Long-Run Revisited

Since the firm in figure 4.6 is making a profit in the short-run, in the long-run it will want to expand both capital and labor and take advantage of constant returns to scale. The firm will therefore expand production by building more factories. Unfortunately for the firm, all other firms in the industry also expand production. Thus, the total supply in the industry increases. This increase in supply will force down the price of the good that the firms

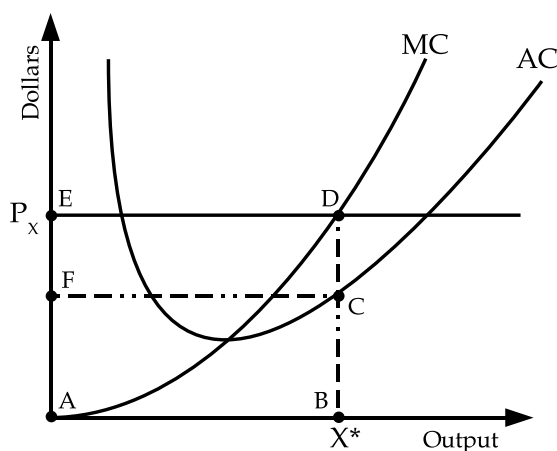


Figure 4.6: The Firm's Output Decision

produce. This decrease in price lowers the profit that each firm earns in each factory.

But if the firm is still making a profit in each factory, it will still want to expand—and so will all the other firms. So in the long-run, firms expand again, supply goes up again, and the price goes down again. This process continues until price equals both average cost and marginal cost. When this happens, profits are zero and firms no longer want to expand. This is the long-run equilibrium.

This equilibrium raises two questions. First, what does zero profit really mean? It means that firms are no longer earning “excess” profits. Included in total revenue is some compensation to the entrepreneur for starting a business. Zero profit just means that the entrepreneur isn't getting a bonus on top of that compensation.

Second, why do firms continue to expand if the end result is zero profit? The answer is that firms do not coordinate their actions. If all firms could agree on fixed output levels, they could stop at a point where they all earn positive profits (OPEC is an example of this type of behavior).

There are two problems with this idea. First, if I know that all of my competitors have agreed not to expand, I have a huge incentive to expand since I can capture the excess profit for myself. It is very hard to enforce the kind of cooperation necessary to maintain positive profits since individual incentives reward not cooperating. (Even OPEC has this problem; member

states often exceed their production quotas and sell their excess oil in secret).

The second problem is that I have assumed firms are price-takers: they are so small that their output decisions don't affect prices. If they are that small, there must be thousands of firms in the industry. It is very difficult to organize such a large number of firms into an effective group. (It is no coincidence that OPEC has few members—many more would make effective coordination difficult).

The critical assumption in this argument is that firms are price-takers. They do not consider the impact their additional output will have on prices because, by assumption, their output doesn't affect prices. Unfortunately for them, their aggregate behavior does affect prices. The result is an equilibrium in which profit equals zero.

## 4.8 Problems

- What two types of cost make up total cost?
  - Give a mathematical expression for average cost.
  - What is the relationship, if any, between marginal cost and total variable cost?
  - Why do we normally assume that marginal cost is increasing?
- Figure 4.7 shows the marginal and average cost curves for a firm that produces grommets. It also shows the market price for grommets,  $P_G$ . (Assume that  $P_G$  is larger than the firm's average variable cost at the optimal output level.)
  - Depict the firm's output level, total revenue, total cost, and profit.
  - Draw the average cost curve for a grommet-producing firm that has the marginal cost curve in the figure above and no fixed costs. Depict your new firm's output level, total revenue, total cost, and profit if it faces price  $P_G$ .
  - Draw a short-run supply curve for the firms in parts 2a and 2b.
- Consider a firm that has a total fixed cost of \$20 and a constant marginal cost of \$5.

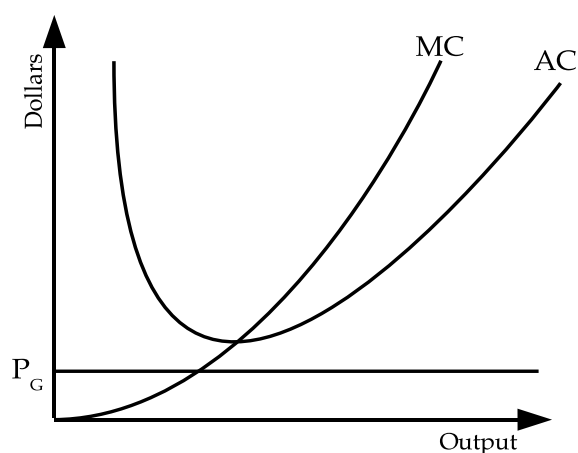


Figure 4.7: The Marginal and Average Cost Curves

- (a) Draw the firm's marginal and average cost curves.
  - (b) What is the smallest price at which the firm will produce output? What is its profit at this price?
4. Define the term “perfectly competitive industry.”

## 4.9 Solutions

1. (a) Total cost is the sum of total variable cost  $TVC$  and total fixed cost  $TFC$ .
  - (b) Average cost is total cost divided by total output  $X$ .
  - (c) Marginal cost is the change in total variable cost that results from a minute change in output.
  - (d) We assume that technology is characterized by diminishing returns. Eventually, adding one more input increases output by less than the previous input. Once diminishing returns have begun, they continue, so that adding inputs yields progressively less additional output.
2. (a) The firm will produce  $X$  units of output, as shown in figure 4.8. It will lose money at this output level, but less money than if it

produced nothing. If it produced nothing, it would lose an amount equal to its fixed costs. At output level  $X$ , its variable costs are less than its revenue, reducing the size of its loss.

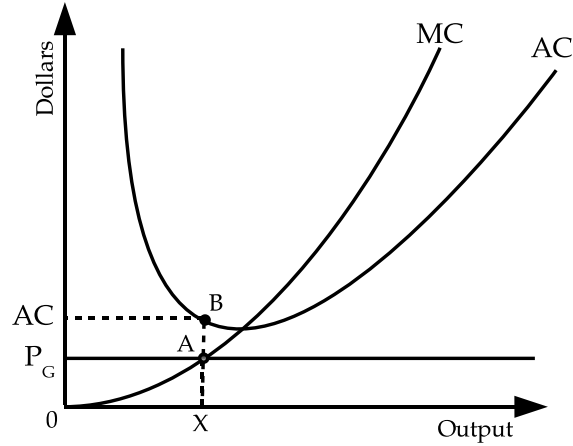


Figure 4.8: The Firm's Profit Maximizing Choice

Total revenue is the price,  $P_G$ , multiplied by  $X$ , the quantity of output. This is the area of the box  $0XAP_G$ . Total cost is  $AC$ , the average cost when output is equal to  $X$ , multiplied by  $X$ , the quantity of output. This is the area of the box  $0XBAC$ . Profit, which will be negative, is the difference between total revenue and total cost. This is the negative of the area of the box  $P_GABAC$ .

- (b) Figure 4.9 shows a firm with no fixed costs. Average cost is always below marginal cost because marginal cost is constantly increasing.

Total revenue is the price  $P_G$  multiplied by  $X$ , the quantity of output. This is the area of the box  $0XAP_G$ . Total cost is  $AC$ , the average cost when output is equal to  $X$ , multiplied by  $X$ , the quantity of output. This is the area of the box  $0XBAC$ . Profit, which will be positive, is the difference between total revenue and total cost. This is the area of the box  $ACBAP_G$ .

- (c) Both firms have the same supply curve: their marginal cost curve. Whether the firm has fixed costs or not, its profit-maximizing output is the output level at which price equals marginal cost.

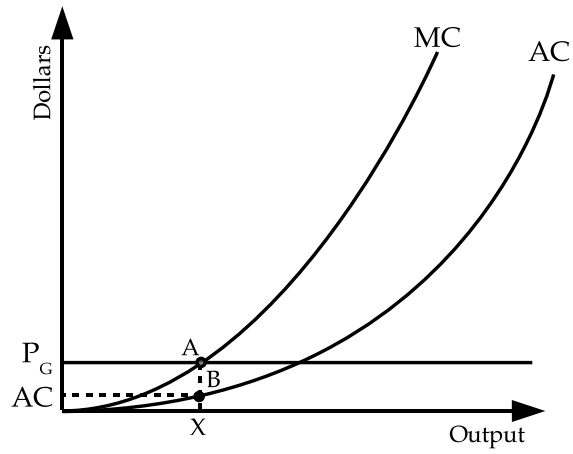


Figure 4.9: The New Firm's Profit Maximizing Choice

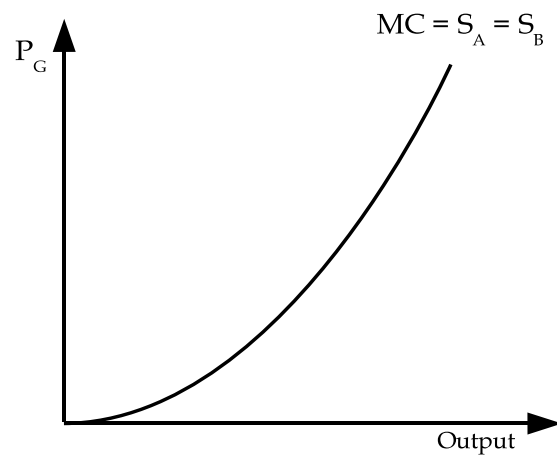


Figure 4.10: The Supply Curve for the Two Firms

3. (a) The firm's marginal cost is always equal to \$5 by assumption, so its marginal cost curve is a horizontal line, as shown in figure 4.11.

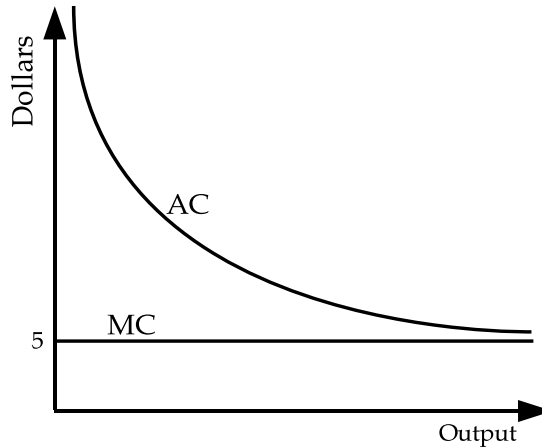


Figure 4.11: The Firm's Marginal and Average Cost Curves

Average cost is total fixed cost over  $X$  plus total variable cost plus  $X$  (see equation 4.7). Total variable cost is always \$5 times  $X$ , so whatever the value of  $X$ , total variable cost over  $X$  equals \$5. Total fixed cost divided by  $X$ , on the other hand, approaches zero as  $X$  approaches infinity. Thus, average cost approaches \$5 as  $X$  gets larger and larger.

$$AC = \frac{TFC}{X} + \frac{TVC}{X} = \frac{\$20}{X} + \frac{\$5X}{X} = \frac{\$20}{X} + \$5 \quad (4.7)$$

- (b) If the price is less than \$5, the firm will produce no output as each unit of output will lose money. When the price is equal to \$5, the firm will produce an infinite amount of output, since each unit of output breaks even due to the constant marginal cost. The firm's profit at this price will be  $-\$20$ , the total fixed cost it incurs. If the price is greater than \$5, the firm will produce an infinite amount of output and earn infinite profit.
4. A perfectly competitive industry is one in which there are many small firms making the same good so that each makes up a miniscule propor-

tion of total output, no firm can affect the price (firms are price-takers), and firms can enter and exit the industry without cost.