Chapter 1

Producers

1.1 The Production Possibilities Frontier

In this chapter I shall use a simple model to examine how a single producer makes supply decisions. I consider a more realistic model in chapter 4, but for the moment I shall assume that there is only one firm in the economy and that it owns land in 5 states. The data presented in table 1.1 describe the productivity of the land in each state.

State	**Corn**	**Wheat**	$-\Delta C/\Delta W$
1	100	50	2
2	40	30	1.33
3	15	45	0.33
4	30	120	0.25
5	250	100	2.5

Table 1.1: Corn and Wheat Production by State

The second column of table 1.1 shows the maximum amount of corn that each state can produce if it grows only corn, the third column shows the same information for wheat, and the fourth column— which was created by dividing column two by column three—shows how many bushels of corn a state must sacrifice to produce one more bushel of wheat. State 1, for example, can produce 2 fewer bushels of corn if it produces 1 more bushel of wheat. This is the cost of producing a bushel of wheat, expressed in terms of lost bushels of corn.

I would like to use the information in table 1.1 to graph the firm's production possibilities frontier (PPF), all the combinations of corn and wheat that the firm can produce. I'll place corn on the Y-axis and wheat on the X-axis. The total row in table 1.1 shows that the Y-axis intercept is at $(0, 435)$ and that the X-axis intercept is at $(345, 0)$. Having graphed the intercepts, I must switch the states one by one from corn production to wheat production. I'll begin by switching state 4, the state whose cost of wheat is lowest (i.e., the state that produces the most bushels of wheat per bushel of lost corn production). I'll then switch the remaining states in ascending order of the cost of wheat. Table 1.2 shows the order in which the states should be switched.

State	**Corn**	**Wheat**	$-\Delta C/\Delta W$
4	30	120	0.25
3	15	45	0.33
2	40	30	1.33
1	100	50	2
5	250	100	2.5

Table 1.2: States Sorted by Cost of One Bushel of Wheat

Table 1.3 shows the changes in wheat and corn production as states are switched from growing corn to growing wheat.

State Switched	**Wheat**	**Corn**
0	0	435
4	120	405
3	165	390
2	195	350
1	245	250
5	345	0

Table 1.3: Transformation from Corn to Wheat

If I graph the points in table three and connect the dots with straight lines, I'll get the production possibilities frontier shown in figure 1.1. The slope of the PPF at any point is the number of bushels of corn *gained* when one more bushel of wheat is produced. This is a negative number because increasing wheat production requires reducing corn production. I can therefore interpret the *negative* of the slope of the PPF as the number of bushels of corn that must be sacrificed to gain one more bushel of wheat (i.e., the cost of wheat.) Notice that the PPF is bowed out outward. This will always be true if the states are switched in the correct order (that is, from lowest cost to highest cost) because doing so ensures that the slope of the PPF decreases as the supply of wheat increases.

Figure 1.1: The Production Possibilities Frontier

The PPF shows us the outer limit of the firm's technology, the best that it can do. Thus, while the firm can produce any of the combinations of wheat and corn on the PPF, it can also produce any combination of wheat and corn that lies inside of the PPF. Interior points are wasteful though, since the firm is capable of producing more of one output without reducing its production of the other. When deciding how much corn and wheat to produce, the firm can therefore ignore all interior points and consider only the points on the PPF. The problem is that there are infinitely many points on the PPF; how is the firm to choose among them?

1.2 Profit Maximization and Prices

I shall assume throughout this book that a firm's only goal is to maximize its total profit. This may not be an accurate description of the complex goals that firms hold, but it is an acceptable simplification and it greatly enhances our ability to analyze firms. The profit that a firm earns is a function of its total output and the prices of the goods that it produces. I can calculate our firm's total profit (π) with the following expression[1].

$$\pi = P_C C + P_W W \tag{1.1}$$

where P_C is the price of corn and P_W is the price of wheat[2]. I can rearrange equation (1.1) to derive iso-profit lines for our firm. An iso-profit line shows, for fixed prices of corn and wheat, all the combinations of corn and wheat output that yield a fixed amount of profit. Equation (1.2) shows the expression for the firms iso-profit lines.

$$C = \frac{\pi}{P_C} - \frac{P_W}{P_C} W \tag{1.2}$$

Equation (1.2) tells us that the Y-axis intercept for each iso-profit line is total profit divided by the price of corn. It also tells us that the slope of each iso-profit line is $-P_W$ over P_C Since I have the slope and Y-intercept of the iso-profit lines, I can graph them.

Let us suppose that the price of corn is \$2 and the price of wheat is \$1 and try to graph the set of the points that yield \$20 in profit. The Y-axis intercept is \$20 (profit) divided by \$2 (the price of corn), or 10 bushels of corn. The slope of the iso-profit line is $-\$1$ (the negative of the price of wheat) over \$2 (the price of corn), so the X-axis intercept must be 20 bushels of wheat. These intercepts make intuitive sense: selling 10 bushels of corn at \$2 a bushel will earn me \$20, as will selling 20 bushels of wheat at \$1 a bushel. We can draw the \$20 iso-profit line on our PPF graph by connecting the Y and X intercepts that we just calculated. If you calculate the profit for any of the points on the resulting line, you will see that producing that combination of corn and wheat will earn \$20 in profit.

Now keep the prices constant and calculate the intercepts when profit is equal to \$40. The Y-axis intercept is 20 bushels of corn (\$40 divided by 2) and the X-axis intecept is 40 bushels of wheat. The slope is still $-\frac{1}{2}$ since the slope depends only on the prices of corn and wheat, which have not changed.

If you plot the \$40 iso-profit line you will notice that it is further from the origin than the \$20 iso-profit line. This is a general principle. As long as prices are positive, the farther an iso-profit line is from the origin, the higher the profit associated with it. To maximize profits therefore, our firm

[1] I'll ignore input costs for the moment

[2] I shall assume that the firm is too small to affect prices

wants to pick the point on its PPF that intersects the highest iso-profit line possible. In most cases, this point will be the point of tangency between the PPF and the iso-profit line, as shown in figure 1.2.

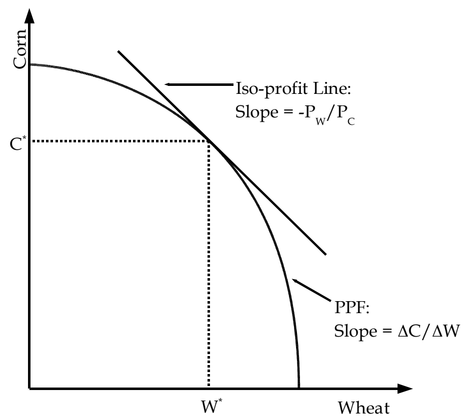

Figure 1.2: Profit Maximization

Figure 1.2 also shows the slope of the PPF and the iso-profit lines. At the point of tangency, these slopes must be equal. That is, at the point of tangency

$$\frac{\Delta C}{\Delta W} = -\frac{P_W}{P_C} \tag{1.3}$$

Equation (1.3) highlights the fact that it is relative prices and relative productivity that matter, not absolute prices or absolute productivity. An increase in the price of corn, if matched by a proportional increase in the price of wheat, will not change the slope of the iso-profit line and will not change the point of tangency (though it will increase total profit!). I can also rearrange equation (1.3) to derive another important interpretation. If I multiple both sides by $-P_C$ I get equation (1.4).

$$P_W = -\frac{\Delta C}{\Delta W} P_C \tag{1.4}$$

The left-hand side of equation (1.4) is the marginal benefit to our firm of producing one more bushel of wheat. It can sell the bushel and earn $\$P_W$. The right-hand side of equation (1.4) is the marginal cost of producing one more bushel of wheat. To produce the extra bushel of wheat it must forgo $-\Delta C/\Delta W$ bushels of corn, each of which is worth $\$P_C$. The product of these two numbers is thus the number of dollars it loses when it produces one extra bushel of wheat.

At the tangency between the iso-profit line and the PPF, the benefit from producing one more bushel of wheat is equal to the cost of producing one more bushel of wheat. If this were not true, the firm would either want to increase (if the marginal benefit was larger) or decrease (if the marginal cost was larger) wheat production. Equation 1.4 therefore provides another perspective on why the profit maximizing output is the point of tangency of the iso-profit line and the PPF.

1.3 The Supply Curve

I can use the PPF and the iso-profit lines to determine our firm's supply curves for wheat and corn. A supply curve shows the relationship between the price of a good and the quantity of that good that is produced. Figure 1.3 provides a typical example.

When I construct a supply curve, I assume that the prices of all other goods are fixed and that the firm's production technology is fixed. In the case of our firm, this means that I fix the price of corn and the productivity of each state. The next step is to pick a price of wheat and find the point of tangency between the PPF and an iso-profit line. This point tells us how much wheat the firm will supply at the price that I picked. Now I pick another price of wheat and go through the same steps. This process yields another price-supply point that I can graph. Repeating this process for all prices of wheat yields a supply curve like the one in figure 1.3 . Do not forget that this supply curve is only valid for the technology and price of corn that I fixed!

If you draw an imaginary PPF and use it to create a supply curve, you will see that an increase in the price of wheat never leads to a decrease in the supply of wheat. It may not lead to an increase in the supply of wheat, but it will never lead to a decrease. This is because I have assumed that firms maximize profits. If the price of corn is fixed and the price of wheat rises,

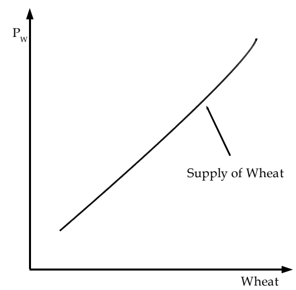

Figure 1.3: The Supply Curve for Wheat

firms will never want to decrease their output of wheat. Indeed, they will typically increase their output of wheat unless they are already producing all wheat and no corn.

1.4 Problems

1. Kate and Mark own a bakery in which they make cookies and bread. Matt can make 20 cookies or 4 loaves of bread in an hour. Kate, on the other hand, can make 10 cookies or 1 loaf of bread in hour. Each can work up to 10 hours a day. The price of a cookie is $1 and the price of a loaf of bread is $5.

   (a) Draw the PPF of the bakery. Indicate the slope and the intercepts.

   (b) Draw the iso-profit line and indicate how many cookies and loaves of bread the bakery will produce in order to maximize profits.

   (c) Now imagine that Kate and Matt acquire a new blender that allows them to produce twice as many cookies and loaves of bread in a day as they did before. Draw the new PPF.

   (d) How many cookies and loaves of bread does the bakery produce after acquiring the new machine?

2. There are two farmers, B and J, who each have a plot of land. B's land is in Montana and J's land is in Kansas. They can each produce wheat

or cattle. If J wants to increase wheat production, he must give up production of 1/100th of a cow per extra bushel of wheat. If B wants to increase wheat production, he must give up production of 1/10th of a cow per extra bushel of wheat.

(a) If 10,000 is the maximum number of bushels of wheat that J can produce on his land, what is the maximum number of cattle that he could raise?

(b) If 1,000 is the maximum number of cattle that B can ranch, what is the maximum bushels of wheat that he could produce?

(c) Draw the PPFs for B and J in separate graphs. Be sure to indicate the slopes on your graphs. (You may assume that one can produce parts of cattle and bushels of wheat in order to get a smooth line.)

(d) If the prices are \$1 per bushel of wheat and \$50 per cow, what will B produce in order to maximize his revenue? What will J produce to maximize his revenue?

(e) What is the slope of the iso-profit lines given the prices in part D?

(f) Now suppose that B and J merge their land. What is the PPF for their combined estate? (Be sure to indicate the slope where necessary!)

3. Bjorn and Vebecka own a bakery. Each works 8 hours a day. Bjorn can make 5 apple danish or 10 cherry danish per hour. Vebecka can make 15 apple danish or 5 cherry danish per hour.

   (a) Draw the PPF for Bjorn and Vebecka's bakery, labeling the slope and intercepts.

   (b) Give non-zero prices for apple and cherry Danish that would make the bakery

      i. produce only apple Danish
      ii. produce only cherry Danish
      iii. produce both apple and cherry Danish.

      Calculate the bakery's total profit for each price.

4. Jenny wants to sell cookies and hot dogs at a baseball game so she hires a bunch of friends to help her (she already has all the equipment and ingredients). Assume that she knows the cooking skills of each friend.

   (a) If all of her friends make hot dogs, they can produce 200 hot dogs. If all of her friends make cookies, they can produce 400 cookies. Draw a production possibilities curve for Jenny and her friends and justify the shape that you drew.

   (b) At the ballpark, hot dogs sell for \$5 and cookies for \$1. Draw some iso-revenue lines on your diagram from part A and label their slope.

   (c) Jenny uses parts A and B to find her profit-maximising combination of hot dogs and cookies. Draw it on your graph.

   (d) If cookies sold for \$2 and hot dogs for \$5, how would Jenny's decision in part C change? Explain.

1.5 Solutions

1. The information in table 1.4 helps to solve this problem.

Individual	**Max Bread**	**Max Cookies**	$-\Delta C/\Delta B$
Kate	10	100	10
Mark	40	200	5
Total	50	300	

Table 1.4: Kate and Mark's Production

   (a) Put C on the Y-axis and B on the X-axis. Then the intercepts are 300 and 50 respectively. Suppose that we begin with both Mark and Kate making cookies. Based on the last column of the table, we want to switch Mark over to bread production first. So from (0,300) to (40,100), the slope of the production possibilities frontier (PPF) is -5. Next, we switch Kate, so the slope from (40,100) to (50,0) is -10.

   (b) The equation for the iso-profit line is

$$\pi = P_C C + P_B B \Rightarrow P_C C = \pi - P_B B \Rightarrow C = \frac{\pi}{P_C} - \frac{P_B}{P_C} B \quad (1.5)$$

so the slope of the iso-profit line is -PB/PC = -5. The trick here is that the slope of the iso-profit line equals the slope of the PPF for the segment from (0,300) to (40,100). So ANY point in this interval is profit maximizing and the iso-profit line and PPF are tangent for the entire segment.

(c) The slope of the PPF doesn't change because $-\Delta C/\Delta B$ hasn't changed for Kate or Mark. The intercepts have changed though, since now both can produce double their old quantity. Thus the new intercepts are C = 600 and B = 100. The slope between (0,600) and (80,200) is -5. The slope between (80,200) and (100,0) is -10.

(d) Since the slope of the PPF hasn't changed and the slope of the iso-profit line (which depends only on the price ratio) hasn't changed, the area of tangency hasn't changed either. Any point between (0,600) and (80,200) maximizes profits.

2. (a) 100 C. We know that J can produce at most 10,000 W and that he must give up 1/100 of C to get 1 W. Thus, he must give up 100 W to get 1 C. Divide 10,000 by 100 and you get 100 C.

(b) 10,000 W. We know that B can produce at most 1,000 C and that he must give up 1/10 of C to get 1 W. If he gives up 1 C therefore he gets 10 W. 1,000 times 10 is 10,000.

(c) Table 1.5 summarizes the relevant information. The slopes are

Individual	**Slope**	**W Intercept**	**C Intercept**
Bill	-1/10	10,000	1,000
Joe	-1/100	10,000	100

Table 1.5: Bill and Joe's Information

$\Delta C/\Delta W$, information that is provided in the question. The information in the question only specifies one value so we can assume that the slope is constant. Hence the production possibilities frontier (PPF) is a straight line. The intercepts are a combination of the values provided in the question and the calculations in parts a and b.

(d) Because -$\Delta C/\Delta W$ is constant, we can make the following calculations: If J produces 1 C, he will earn \$50. Instead of producing 1 C, he can produce 100 W, yielding \$100. Hence he will produce all W. Since his maximum is 10,000 W, he will produce 10,000 W. If B produces 1 C, he will earn \$50. Instead of producing 1 C, he can produce 10 W, yielding \$10. Hence he will produce all C. Since his maximum is 1,000 C, he will produce 1,000 C.

(e) The equation for the iso-profit lines provides the answer:

$$\pi = P_C C + P_B B \Rightarrow P_C C = \pi - P_B B \Rightarrow C = \frac{\pi}{P_C} - \frac{P_B}{P_C} B \quad (1.6)$$

so the slope is $-P_W/P_C$, which is -1/50.

(f) The intercepts are 1,100 C (since B and J can make 1,000 and 100 C at most respectively) and 20,000 W (since B and J can each make 10,000 W at most). Suppose that we begin with both B and J producing all C. If we must switch one into wheat production we will choose J, since

$$-\frac{\Delta C}{\Delta W_J} = \frac{1}{100} < \frac{1}{10} = -\frac{\Delta C}{\Delta W_B} \quad (1.7)$$

So the slope between (0,1100) and (10000,1000) is -1/100; elsewhere it is -1/10.

3. Table 1.6 helps to solve this problem.

Person	**Apple**	**Cherry**	**$-\Delta$A/ΔC**
Bjorn	40	80	1/2
Vebecka	120	40	3
Total	160	120	

Table 1.6: Bjorn and Vebecka's Production

(a) The Y-intercept is 160, the X-intercept is 120, the slope is -1/2 from (0,160) to (80,120) and -3 from (80,120) to (120,0).

(b) Figure 1.4 depicts the new PFF.

i. If -P_C/P_A is greater than -1/2, the bakery will produce only apple danish. The bakerys profit will be PA times 160.

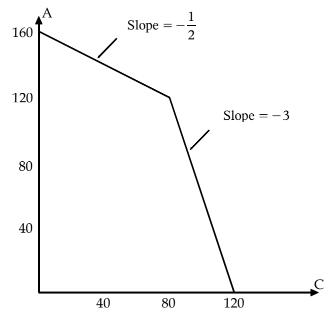

Figure 1.4: The Bakery's New PPF

ii. If -P_C/P_A is less than -3, the bakery will produce only cherry danish. The bakerys profit will be PC times 120.

iii. If -P_C/P_A is less than -1/2 and greater than -3, the bakery will produce both apple and cherry danish. Bjorn will make the cherry danish and Vebecka will make the apple danish. The bakerys profit will be PA times 120 and PC times 80.

4. Let C denote cookies, H denote hot dogs, and P_C and P_H be their respective prices.

   (a) The PPF in figure 1.5 shows all the combinations of cars and computers that can be produced from a fixed set of inputs. It is bowed outward because of specialization. If none of the inputs is specialized, the PPF will be a straight line. If the inputs are specialized, they can always be switched in an order that results in the PPF being bowed outward (i.e., switch first the inputs that are most specialized to the task to which they are being switched). The diagram below illustrates parts A, B and C.

   (b) The slope of the iso-revenue lines is -1/5. This comes directly from the equation for a family of iso-revenue lines.

   (c) Point Z, the tangency between the PPF and the highest possible iso-revenue line, is Jenny's profit maximizing point.

   (d) If the price of cookies rises to $2, the slope of the equation for a

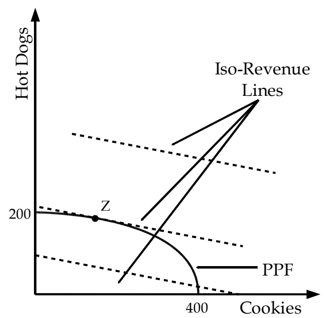

Figure 1.5: Jenny's PPF and Iso-Revenue Lines

family of iso-revenue lines changes to

$$R = P_C C + P_H H \Rightarrow H = \frac{R}{P_H} - \frac{P_C}{P_H} C = \frac{R}{5} - \frac{2}{5} C \qquad (1.8)$$

The new slope is thus -2/5. This increase in the (absolute value of the) slope of the iso-revenue lines makes them steeper. This shifts the tangency point to the right and increase Jenny's production of cookies, as shown in figure 1.6. Figure 1.7 shows Jenny's supply curve for cookies, which is upward sloping.

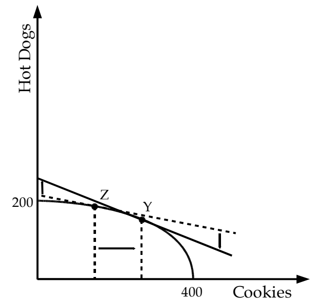

Figure 1.6: Jenny's New Iso-Revenue Lines

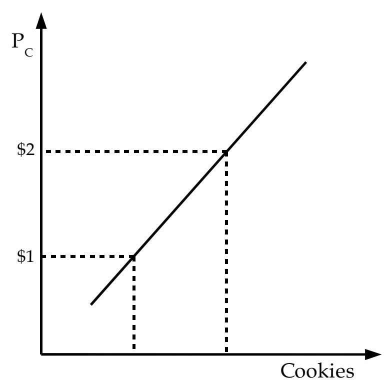

Figure 1.7: The Supply Curve for Cookies