Relationship To Cohen's Work

Why This Orthonormal Space? One rationale for the orthonormalized color matching functions is intuitive. If detection by cones loses information, and the goal of color technology is to stimulate color vision in its 3 independent dimensions, then color stimuli should be evaluated along truly independent dimensions. Another benefit is that it gives a new specific meaning to tristimulus values. If **A** is a matrix with *any* set of color matching functions as its columns, such as perhaps $\mathbf{A} = [\,|\bar{x}\rangle, |\bar{y}\rangle, |\bar{z}\rangle]$, then the projector matrix **R** can be found by[7]

$$\mathbf{R} = \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}} \,. \qquad (7)$$

Matrix **R**, discovered by Jozef Cohen, is called a projector matrix because it extracts from a spectral radiance $|N\rangle$ that component that lies in the vector space of the color matching functions, called $|N^*\rangle$:

$$|N^*\rangle = \mathbf{R}|N\rangle \,. \qquad (8)$$

In words, $|N^*\rangle$ is the fundamental metamer of $|N\rangle$. **R** is invariant to the choice of color matching functions in **A**. They could be cone sensitivities, or they could be $\boldsymbol{\Omega}$. If we let $\mathbf{A} = \boldsymbol{\Omega} = [|1\rangle, |2\rangle, |3\rangle]$, so the columns are the orthonormal color matching functions,

$$\mathbf{R} = \boldsymbol{\Omega}(\boldsymbol{\Omega}^{\mathrm{T}}\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}^{\mathrm{T}} \,. \qquad (7)$$

Thanks to orthonormality, the expression $\boldsymbol{\Omega}^{\mathrm{T}}\boldsymbol{\Omega}$ is the 3×3 identity matrix. Its inverse is also an identity matrix, and **R** reduces to

$$\mathbf{R} = \boldsymbol{\Omega}\boldsymbol{\Omega}^{\mathrm{T}} \qquad \text{<only for orthonormal cmf's!>} \quad . \qquad (8)$$

Keeping in mind the definition of $\boldsymbol{\Omega}$ and the rules of matrix multiplication, this can be written in an alternate form,

$$\mathbf{R} = \sum_{j=1}^{3} |\,j\rangle\langle j| \quad . \qquad (9)$$

The notation of Eq. (9) is sometimes used for what is called a unity operator. In any case, Eq. (9) then leads to

$$|N^*\rangle = |1\rangle\langle 1|N\rangle + |2\rangle\langle 2|N\rangle + |3\rangle\langle 3|N\rangle \,. \qquad (10)$$

Each complete bracket, such as $\langle 1|N\rangle$, is an inner product, and as such is a single number. These brackets are the coefficients in the approximation of $|N\rangle$ by a basis function expansion. They are also the tristimulus values of $|N\rangle$ in the orthonormal system. This fact gives a level of meaning to tristimulus values that they otherwise lack, and Eq. (9) can aid in deriving little working formulas for chores such as converting a tristimulus vector from one basis to another[1].