How White Light Works
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Suppose that two lights have about the same chromaticity, but comprise different sets of narrow-band colors. Each light's component colors, according to the power at each wavelength, map to vectors in color space, which add to give the light's tristimulus vector. Decomposing the lights vectorially leads to a picture of how they differ, and how they will function when shined on colored objects. While the XYZ system can be used for vector operations, it is not well suited to vector diagrams and discussion. Using three orthonormal color-matching functions maps colors into Jozef Cohen's more logical color space, which is free of arbitrary elements. The particular functions are a slightly arbitrary choice, but have a logic of their own and give intuitive meaning to the axes. In Cohen's space, the rationale of RGB primaries such as television phosphors becomes obvious, since they are in regions where unit power gives the longest vectors. When a light of poor color rendering is analyzed, the chain of component vectors takes a shortcut to the white point, rather than progressing to green and back towards red. Three-band lights typically hit the long-vector wavelengths to give a slight enhancement of red-green contrast. A related publication is James A. Worthey, "Color matching with amplitude not left out," *Proceedings of the 12th Color Imaging Conference*, Scottsdale, AZ, USA, November 9-12, 2004, http://www.imaging.org . One color rendering example with a 3D graphic is on http://www.jimworthey.com . The Orlando presentation will emphasize color rendering, with examples to include LED lighting. In summary, the vectorial approach demystifies color rendering and presents it as an interesting topic for science and engineering. Three simple functions embody lifetimes of work by Cohen, Thornton, Brill, Worthey and others.

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Color is often discussed in the same repetitive terms. A light's spectral power distribution multiplies three standardized functions, $\bar{x}, \bar{y}, \bar{z}$, then those products are summed. For example, $Y = \sum_{\lambda=360}^{830} L_\lambda \bar{y}_\lambda$ and so forth, then x = X/(X+Y+Z), then the ordered pair (x,y) is plotted in the chromaticity diagram. The chromaticity diagram is then good for some tasks, such as specifying colored signal lights. Chromaticity (x,y) tells the signal light's direction in color space, independent of its intensity. Other applications, such as television phosphors for example, intrinsically call for colored lights to be generated *and then mixed*. Although the textbooks will not tell you this[1], the red, green and blue TV phosphors are distinguished by their strong action in mixtures [2]. Colored objects under a white light can only reflect the colors available from the light, then those colors mix. Through computational experiments and other research, Thornton identified the strongly acting wavelengths, which he called Prime Colors [2,3,4,5].

To get beyond XYZ, we can look at the same facts in alternate forms. For example, the functions of Fig. 1 are sensitivities for the retina's 3 cone types, but they are equivalent to $\bar{x}, \bar{y}, \bar{z}$ of the 2° observer in the color matches that they predict. A key feature of human vision
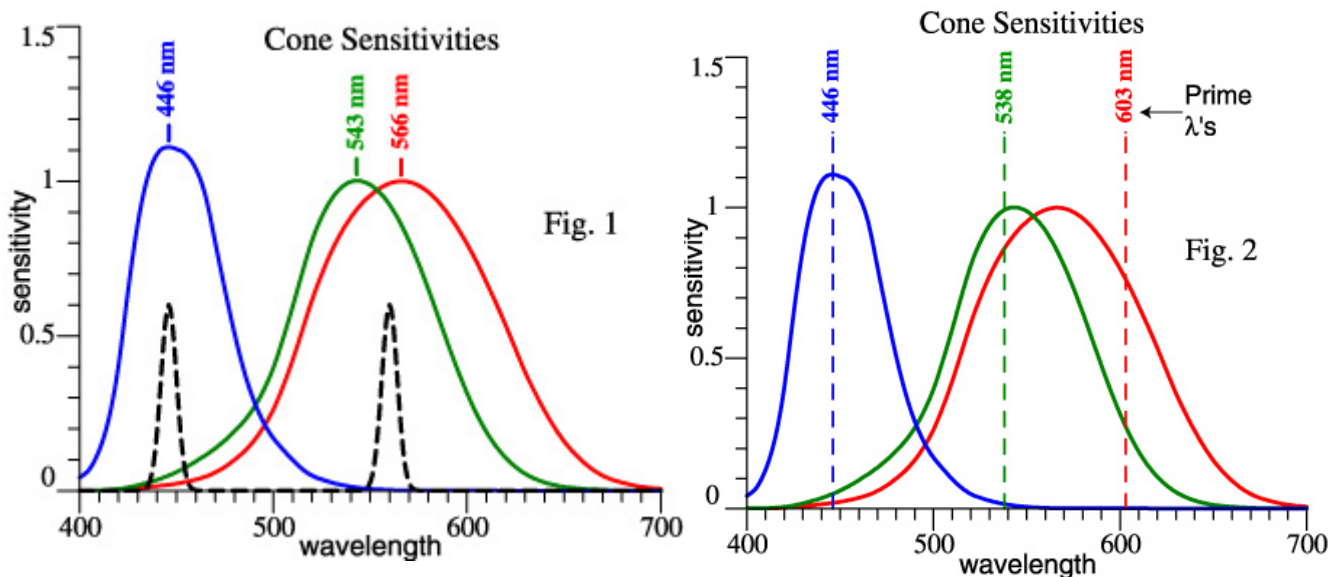
Fig. 1. Cone sensitivities consistent with the 2° observer. Wavelengths of the function peaks are indicated.
Fig. 2. Same cone sensitivities. Dashed lines indicate the Prime Color wavelengths.

is the high degree of overlap of the red and green functions. As one measure of overlap, the red sensitivity peaks at 566 nm, in the yellow part of the spectrum. Fig. 1 by itself offers a lesson about color rendering. A white light is one that stimulates all three cone types. Because of overlap, a narrow band in the yellow part of the spectrum stimulates both red and green cones. Add a narrow band in the blue, and you have a white light comprising two narrow bands, as shown by the dashed line. But such a light loses reds and greens, reducing all objects to a palette of blue, white, yellow, and so forth[2].

Red, yellow, and green lights are distinguished by their relative effects on red and green cones. The wavelengths that act strongly in mixtures will then be the ones where the overlapping functions are *the most different*. In Fig. 2, the cone sensitivities appear again, with vertical dashed lines that indicate the Prime Color wavelengths. The gaps between the functions are indeed large at the prime wavelengths, though Thornton's most refined analysis was used for the actual derivation of the prime wavelengths[5, 6].

Jozef Cohen's Color Space and the Orthonormal Basis

A version of opponent colors will help us to confront overlapping sensitivities. That is, we consider a sum **red + green** as one "signal," and the difference **red – green** as a second independent signal. The retina does such a transform before image data reach the optic nerve, and color television also uses signals organized as white-black, red-green and blue-yellow. The following set of color functions will prove helpful for working with color stimuli:

1. The first function, $\omega_1(\lambda)$, is proportional to the usual $\bar{y}(\lambda)$, but re-normalized so that its summed-square value = 1. This "achromatic" function is in fact a sum of red and green sensitivities, $\bar{y} = 0.6372r + 0.3924g$. That is, red and green cones add to give a whiteness function.

2. The second function, $\omega_2(\lambda)$, is the opponent combination **red – green**, with the coefficients chosen so that it is orthogonal to $\omega_1(\lambda)$. That is,

$$\sum_\lambda \omega_1(\lambda)\omega_2(\lambda) = 0 \qquad . \tag{1}$$

   Function $\omega_2(\lambda)$ is also normalized so that its summed-square = 1.

3. The third function, $\omega_3(\lambda)$, is a combination of all 3 cones, such that it is orthogonal to the other 2 functions, and again normalized. In the end, then, we have a set of orthonormal functions which are also an opponent-color set, Fig. 3.
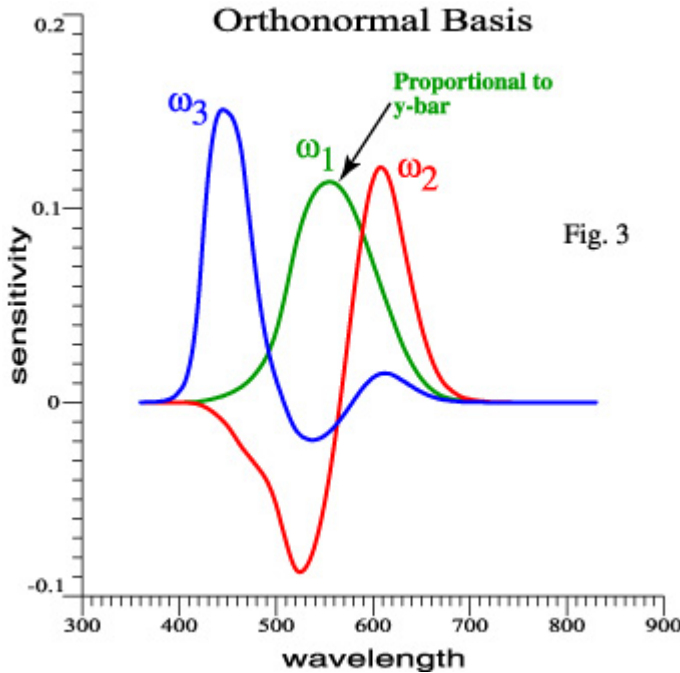

Fig. 3

If a matrix Ω has the functions ω_j as its columns,

$$\Omega = [\omega_1(\lambda) \quad \omega_2(\lambda) \quad \omega_3(\lambda)] \quad , \tag{2}$$

then for a light L the tristimulus vector V, which does the work of X, Y, Z is given by

$$V = \Omega^\mathsf{T}L \quad . \tag{3}$$

Here L is the spectral distribution in column matrix form. The matrix product operation *is* the expected sums over λ. The 3 elements of V are independent measures of the light L, meaning that each number is as informative as possible. The first value, v_1, has the intuitive meaning of whiteness, while v_2 is redness (if > 0) or greenness (if < 0). The last value, v_3, is blue versus yellow.

Fig. 3. The orthonormal color matching functions.

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Fig. 4

Fig. 4. The orthonormal color matching functions have now been combined to give a single curve in 3-dimensional space. The curve is in fact Cohen's locus of unit monochromats, but drawn with axes determined by the choice of basis functions.
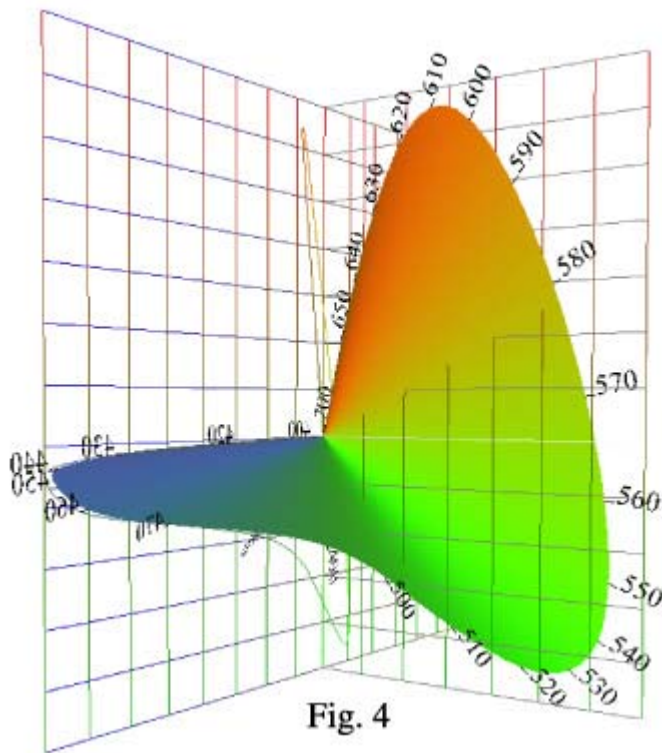
So, these are simple and appealing ideas, that the vector components should be independent measures of the light, with intuitive color names on the axes of color space. A deeper meaning is found by reference to Jozef Cohen's work. Cohen showed that the facts of color matching imply vector relationships among color stimuli[7,8]. Those relationships do not depend on the color matching functions that one starts with, whether $\bar{x}, \bar{y}, \bar{z}$, the cone functions of Fig. 1, or the set Ω, Fig. 3. In any case, the vector relationships among monochromatic (narrow band) lights are summarized by a curve in 3 dimensions, which Cohen called "the locus of unit monochromats." [7,8] Combining the orthonormal functions into a 3-dimensional plot gives the same locus of unit monochromats, but now plotted in relation to specific axes.

Cohen's algebra emphasized the projector Matrix **R**, from which the locus of unit monochromats and other results can be found. For given color-matching data, say the CIE 2° observer, **R** is a completely fixed array of numbers. That simple fact makes clear the invariance of things that depend on **R**, such as the vector relationships among color stimuli. The orthonormal basis is further discussed in Reference 9 and in related materials on the author's web site, http://www.jimworthey.com .

Working Class Summary

Someone may ask, "So you are selling a new set of color matching functions. You start with cones and create an orthonormal set, and that is interesting. But in the end, what do you have that is new?" If we consider $\bar{x}, \bar{y}, \bar{z}$, as the "old" functions, then ω_1 is a multiple of $\bar{y}$, so it is not new. The third orthonormal function, ω_3, is really a blue function with a little extra variation, therefore it is similar to the old $\bar{z}$. So, for practical purposes, 2/3 of the "new" system is similar to the old system. The remaining old function is $\bar{x}$, an arbitrary magenta primary. The orthonormal system replaces $\bar{x}$ with the red-green opponent function, ω_2, allowing the axes to have intuitive meanings, namely white, red or green, and blue or yellow. Any set of color-matching functions can map stimuli to vectors. The orthonormal feature of the new cmf's spreads out the vectors as much as possible.

Application to Color Rendering

The idea of "unit monochromats" is that a vector in color space is found for each light of unit power at a specific wavelength λ. Examples would be one-watt lights at 400 nm, 401 nm, and so on. Then Fig. 4 shows two ideas that we know intuitively: different wavelengths map to

different directions in color space, and they also map to different amplitudes. For the first time we have a scheme for clear talk about color vectors:

1. The orthonormal basis spreads out the vectors and plots independent stimuli at right angles.
2. The orthonormal basis gives a non-arbitrary scaling. To make a long story short, the element of *V* are scaled to the stimulus [9].
3. Axes have intuitive meaning.

A basic starting point for a color rendering calculation is to have two or more lights that map to the same tristimulus vector—the same color and intensity—but have different spectral power distributions. For example, let the lighting change from daylight to high-pressure mercury light [10]. The tristimulus vectors are: daylight = [3.751, 0.03474, 1.396] ; mercury light = [3.751, 0.03204, 1.389]. The arrows in Figures 5 and 6 are based on 64 Munsell papers as measured
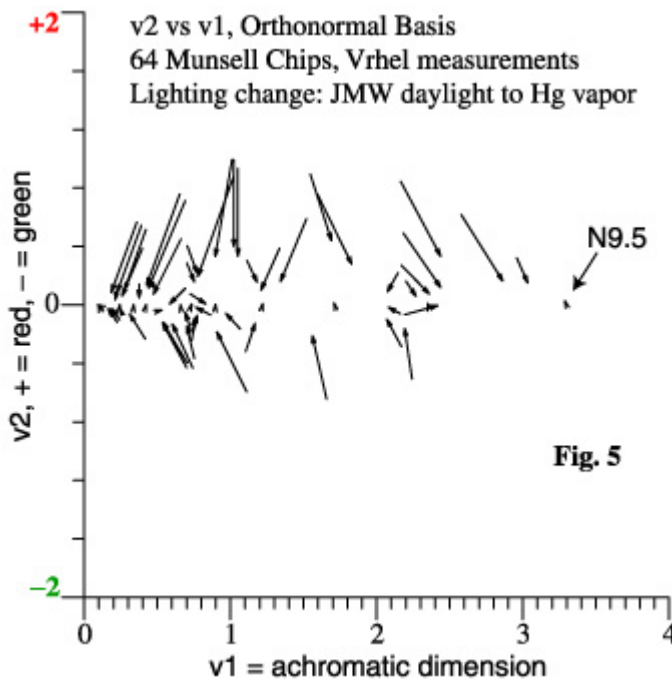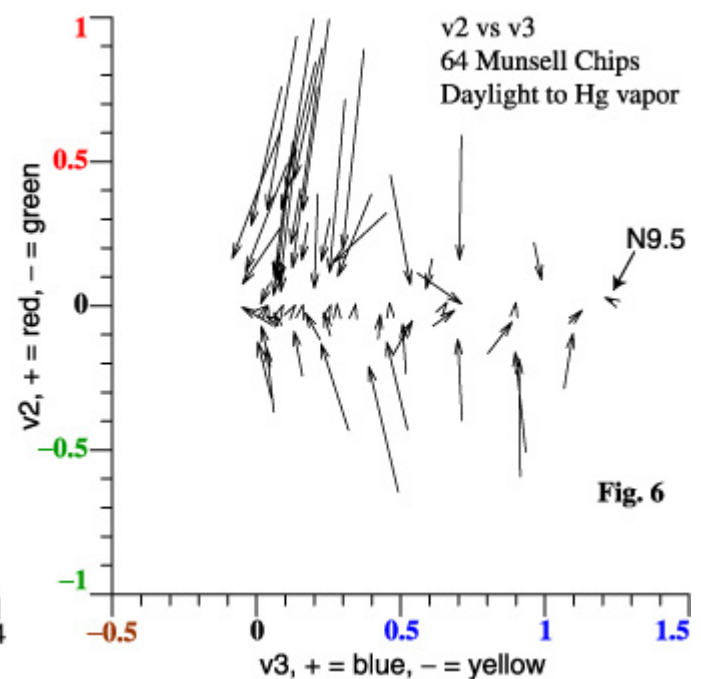

Fig. 5. A color rendering example is presented in the new coordinates—essentially Cohen's color space. The 64 Munsell chips measured by Vrhel et al. are considered to be viewed first under daylight and then under a high-pressure mercury light of the same tristimulus vector. The lighting change moves the colors towards neutral in the red-green dimension, v2. As a point of reference, the lightest neutral paper is indicated by it Munsell notation, N9.5 .

Fig. 6. Another view of the same arrows indicating Munsell papers seen first under daylight and then under high-pressure mercury light. Again the whitest paper, N9.5, is pointed out as a reference.

by Vrhel et al [11]. The tail of each arrow is the stimulus vector of a paper under daylight, while the head is for the same paper under the mercury light. As a point of reference, the lightest Munsell paper is indicated by its notation, N9.5 . The mercury light acts much like the hypothetical 2-bands light in Fig. 1, dulling reds and greens, pulling them towards neutral in the red-green dimension. These figures are based on detailed calculation from lights and reflectances. The only novel element is the new color space—Cohen's space with intuitive axes. The overall picture is simplified because the lights are neither red nor green, causing the neutral papers to line up perpendicular to the red-green axis. Other normal lights could have redness. The tiniest arrows indicate the neutral papers; gray is gray under both lights. Notice that the actual color vectors are points, or if you wish they are arrows from the origin.

The indicated arrows are difference vectors *V*(mercury vapor) – *V*(daylight). Figs. 5 and 6 show two orthographic projections of facts in a 3-space. An alternate presentation on a computer screen uses a virtual reality viewer and allows the user to rotate the scene. For example, Fig. 4 is a screen grab of a virtual reality graph.[12]

Components of a White Light

Eq. (3) expresses *V* by a matrix product. In the prototypical lighting situation, one light *L* shines on a number of surfaces. To emphasize that the color stimulus is a 3-vector, the stimulus vector of one surface s_i can be written

$$V_i = \begin{bmatrix} \langle \omega_1 L | s_i \rangle \\ \langle \omega_2 L | s_i \rangle \\ \langle \omega_3 L | s_i \rangle \end{bmatrix} \quad \text{or} \quad V_i = \begin{bmatrix} \langle \omega_1 L | \\ \langle \omega_2 L | \\ \langle \omega_3 L | \end{bmatrix} | s_i \rangle \quad . \quad (4)$$

A bra, such as $\langle \omega_1 L|$ is a row vector, while a ket such as $|s_i\rangle$ is a column vector, so that the complete bracket $\langle | \rangle$ denotes an inner product, a single number. The stimulus vector is three inner products, or in the right-hand version things are factored out to emphasize the vector containing the three functions $\omega_j L$, which are functions of wavelength $\omega_j(\lambda)L(\lambda)$. These functions, the "object color matching functions," [13] combine the one light and the three cmf's, which apply for all surfaces.

Taken as a unit, the 3×1 matrix is *a vector function of wavelength*. It could be graphed as a kind of distorted locus of unit monochromats, similar to Fig. 4. Let us have that concept without making the graph. At each wavelength, the vector function points in a certain direction, with a certain amplitude. If all those vectors are added, vectorially, tail-to-head, the result is the tristimulus vector of the light. To add all the vectors in one step, that would be a re-statement of Eq. (3), not interesting. Instead, let us add the vectors a few at a time, creating little color vectors which can be chained tail to head, to give the total vector. Then present the vector chain graphically. The example above referred to a daylight and a mercury vapor light with equal tristimulus vectors, in other words color-matched lights. The spectral distributions are shown in Fig. 7. The vertical dashed lines indicate boundaries for chopping the spectra into narrow bands. The bands are 10 nm wide except at the ends of the spectrum. The color vector for each band can be computed, leading to a 3-dimensional graph, represented by a screen grab in Fig. 8. The arcing chain of thin arrows (near the bottom) shows how the color components of daylight add to give its total stimulus vector. The red and green components balance out. The mercury light
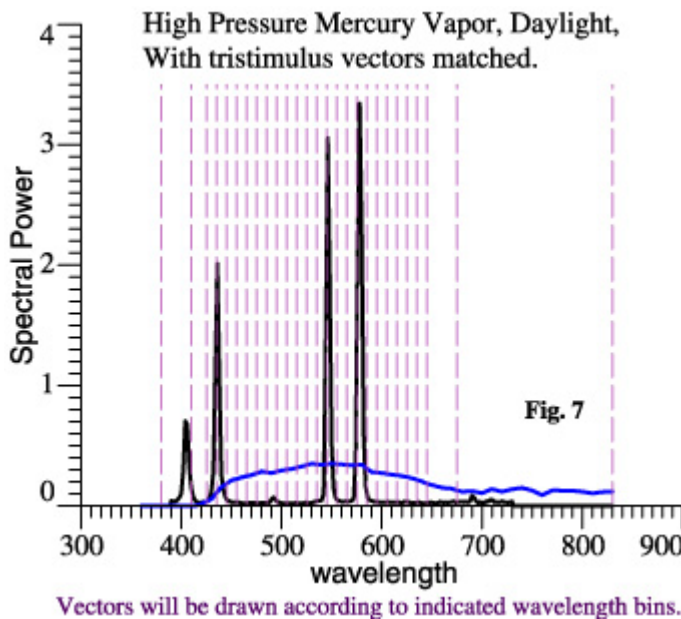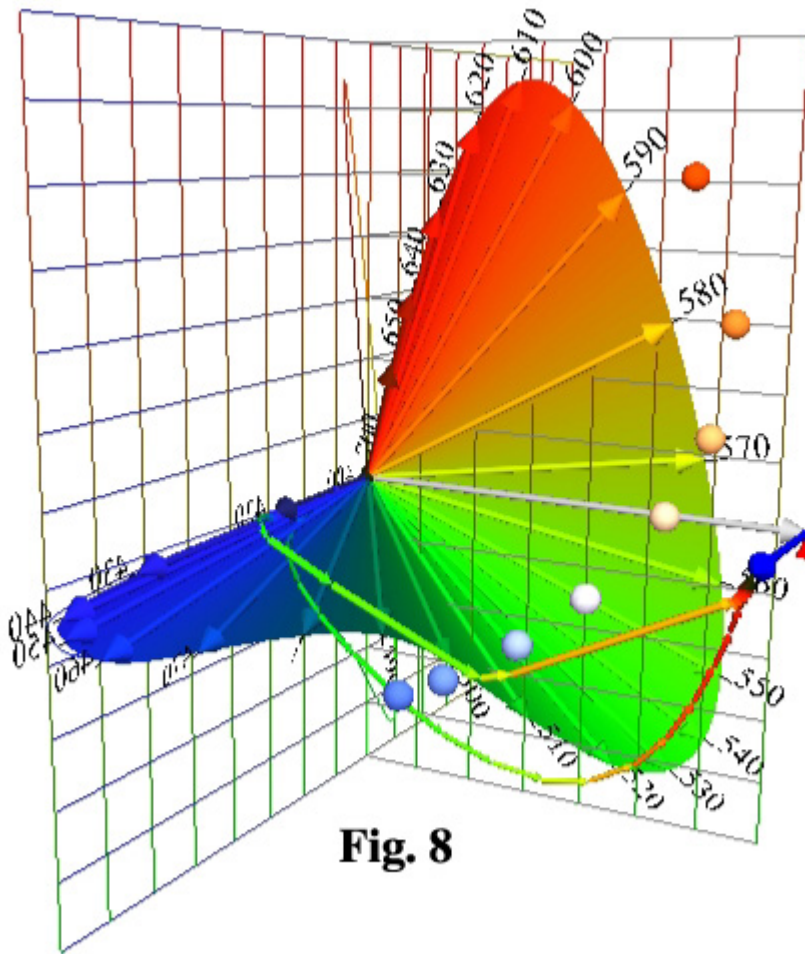

High Pressure Mercury Vapor, Daylight, With tristimulus vectors matched.

Fig. 7

Vectors will be drawn according to indicated wavelength bins.

Fig. 7. The color-matched mercury light (with spikes) and daylight (smoother), chopped by dashed lines.

packs most of its power into a few narrow bands; thus, a few long vectors contribute much of

Fig. 8. Screen grab of a virtual reality graph. The locus of unit monochromats (Fig. 4) is shown for reference. The white axis points to the right, while red is up and green is down. Spheres show the blackbody locus at constant radius. The components of daylight add in a smooth arc. Mercury light components take a shortcut to the white point, with less deviation to green and back to red.

the vectorial total. Although the mercury light's chain also arcs across the bottom of the picture, it has less red and less green. The red and green balance out, but there is less of each. Red and green objects (like bell peppers, for example) depend on the actual red and green components to be seen in vivid color.

A third chain of 3 arrows also sums to the same point. In color they are gray, red, and blue. The gray arrow is right along the achromatic axis, while the red arrow is quite short and appears as an arrowhead only. The 3 components represent the total stimulus, the same information usually written as [*X Y Z*]. In the usual discussion, the 3 numbers are calculated, and it is then assumed that all the information from colorimetry has been used up. A drawing like Fig. 8 compares lights on the basis of their component colors. The component vectors are found by simple colorimetry, just as total stimulus vectors are routinely calculated.

LED Examples

The spectral distributions of two white LEDs were found in a vendor data sheet [14]. Fig. 9 shows the spectral power distribution of a "5500 K white LED," and a matching daylight SPD for comparison. By my calculation, the LED's color temperature is 6115 K. The stimulus vectors are LED: [3.234, 0.1824, 2.347] ; JMW daylight: [3.233, 0.1832, 2.349] . From Fig. 9, we can say in general terms that the LED is rich in yellow and poor in red and green. The chains of component colors confirm this idea. The chain which has slightly fatter arrows and takes more of a shortcut to the final point is the LED. Once again, the right-angle components of the LED's stimulus vector (numbers right above) are shown by the fattest arrows and are equivalent to the usual tristimulus values of the light. The spheres indicate the blackbody locus at constant radius, indicating temperatures of 2000, 3000, 4000, 5000, 7000, 1e+004, 2e+004, 1e+005 K.

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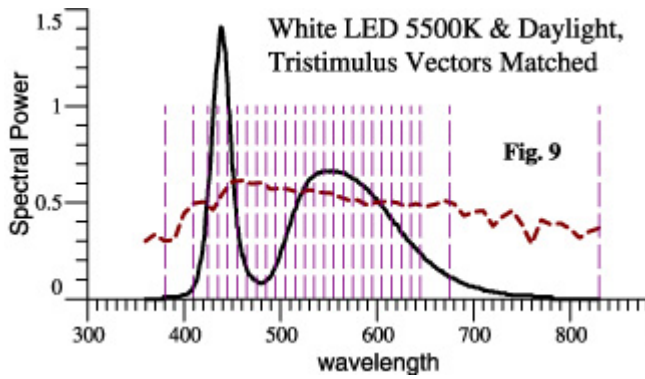
Fig. 9. Spectral distribution of a "5500 K" white LED and a version of JMW daylight with the same stimulus vector.

Fig. 10. The lights of the previous figure are now compared by looking at their component colors. The chain of slightly fatter arrows shows that the LED has more yellow, less red and green.
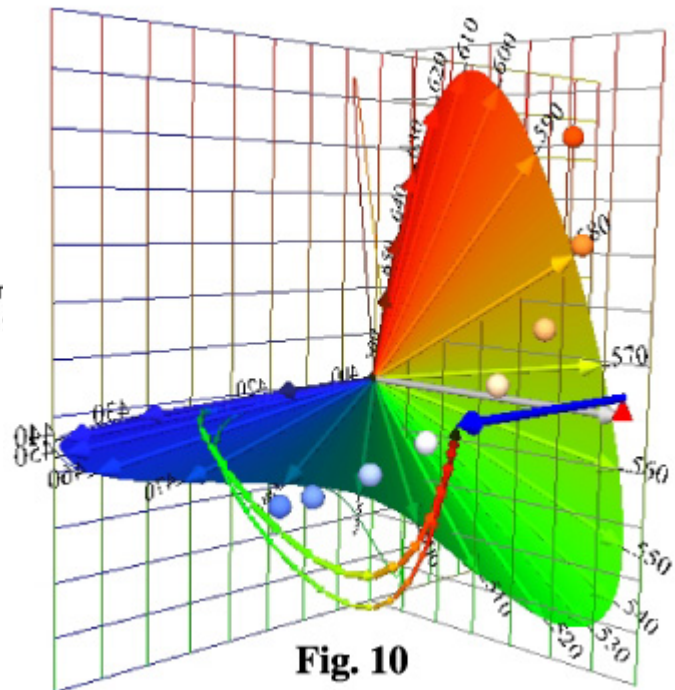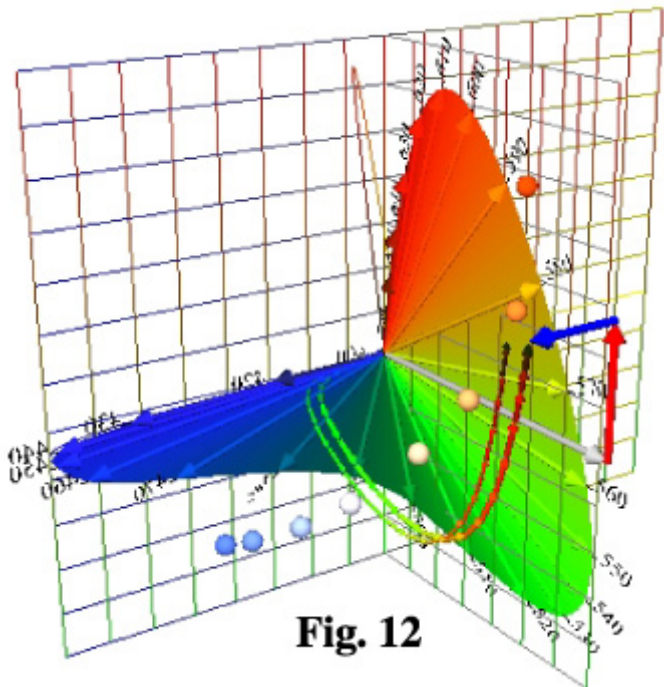
The other white LED is a warmer color. I find the correlated color temperature = 3300 K.
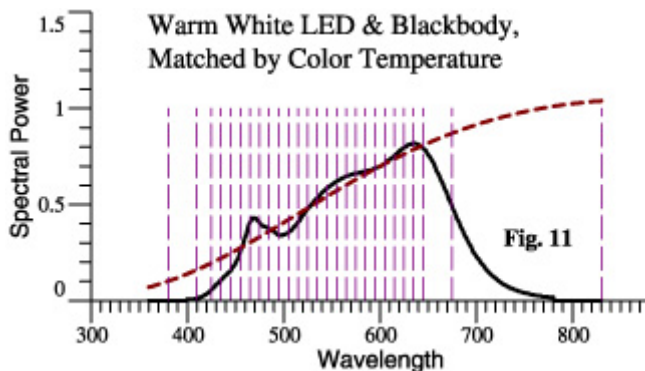
Fig. 11. Spectra of warm white LED and blackbody matched by CCT.
Fig. 12. Same 2 lights, compared by component colors. The stimulus vectors are not quite the same.

In one simple scheme, the orthonormal color space applies major results of Thornton, Cohen, and Worthey and important ideas from M. H. Brill and S. L. Guth and others. Details have been published [9] and will hopefully be further explained in another new article. Reference [2] gives explanatory background. See my web site, http://www.jimworthey.com .

In brief summation, use Guth's 1980 model, apply his formulas with the usual $\bar{x}, \bar{y}, \bar{z}$. Keeping the sequence achromatic, red-green, blue-yellow, orthonormalize the functions by Gram-Schmidt. The resulting color space is then exactly that of Jozef Cohen, graphed with specific axes. The color space is appropriate to all discussions of color mixtures, primary colors, and

so on, not only color rendering. On the other hand, use of vectorial color shows that color rendering is an interesting part of colorimetry: how white lights work. Combining the orthonormal functions draws Cohen's "locus of unit monochromats." The extreme points of that locus are *approximately* Thornton's Prime Colors. Any set of color matching functions describes action in mixtures, so vector amplitude is "strength of action." The amplitude of a stimulus vector in the orthonormal scheme is the same as what Cohen called the amplitude of the fundamental metamer, so it is not arbitrary. The fact of prime colors—that certain wavlengths are most distinctive in color mixing—is intrinsically a part of the method of color components, Figs. 8, 10, and 12.

The goal of this paper has been to demystify color rendering. The vector method is not based on a hidden assumption about what is important. The emphasis is on a detailed display of facts. Such plots can be a working tool for lighting inventors, and a means for explaining a product. A vector plot is not especially helpful in studying object metamerism, but the orthonormal functions would work well in the opponent-based metamerism method [13].

References:
[1] R. W. G. Hunt. 1975. The Reproduction of Color. King's Langley, England. Fountain Press.
[2] James A. Worthey. 2003. "Color rendering: asking the question," *Color Res. Appl.* **28**(6):403-412.
[3] William A. Thornton. 1971. "Luminosity and color-rendering capability of white light," *J. Opt. Soc. Am.* **61**(9):1155-1163.
[4] William A. Thornton. 1972. "Three-color visual response," *J. Opt. Soc. Am.* **62**(3):457-459.
[5] Michael H. Brill, Graham D. Finlayson, Paul M. Hubel, William A. Thornton. 1998. "Prime Colors and Color Imaging," Sixth Color Imaging Conference: Color Science, Systems and Applications, Nov. 17-20, 1998, Scottsdale, Arizona, USA. Springfield, Virginia. IS&T.
[6] Michael H. Brill, James A. Worthey. 2005. "Color Matching Functions When One Primary Wavelength is Changed." This article has been submitted for publication, but for now is available as a preprint: http://www.jimworthey.com/cmfchange1primary.pdf .
[7] Jozef B. Cohen, William E. Kappauf. 1982. "Metameric color stimuli, fundamental metamers, and Wyszecki's metameric blacks," *Am. J. Psych.* **95**(4):537-564.
[8] Jozef B. Cohen. 2000. Visual Color and Color Mixture: The Fundamental Color Space. Champaign, Illinois. University of Illinois Press.
[9] James A. Worthey. 2004. "Color matching with amplitude not left out," Proceedings of the 12th Color Imaging Conference: Color Science and Engineering, Scottsdale, AZ, USA, November 9-12, 2004. Springfield, Virginia. Published by IS&T, http://www.imaging.org .
[10] High-pressure mercury light was measured by Worthey in 1985.
[11] Michael J. Vrhel, Ron Gershon, and Lawrence S. Iwan, 1994. "Measurement and analysis of object reflectance spectra," *Color Res. Appl.* **19**, 4-9. The data were obtained in digital form by ftp from server ftp.eos.ncsu.edu in directory /pub/spectra. Data recently appeared on the same server in directory /pub/eos/pub/spectra/ .
[12] The virtual reality language is VRML, Virtual Reality Modeling Language. A free viewer, the "Cortona VRML Client," is available from http://www.parallelgraphics.com .
[13] James A. Worthey. 1988. "Calculation of metameric reflectances," *Color Res. Appl.* **13**(2):76-84 (April 1988).
[14] Lumileds. 2005. Technical Data Sheet DS23. Found at http://www.lumileds.com .