Tuesday, 2004 November 2, 16:05:47
JAW, Wednesday, 2004 October 6, 23:34:42

Color Matching with Amplitude Not Left Out

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Scottsdale, Arizona, Color Imaging Conference 12
Friday, 2004 November 12, 11:20 am

Before I explain the details, let me list the 3 main ideas.

- Number one is a set of orthonormal color matching functions.
- Number 2 is a graph in 3 dimensions of the eye's vectorial sensitivity to colored lights. Combining the orthonormal functions generates this graph.
- Idea number 3, in short, is tristimulus vectors, and especially their amplitudes.

As color is often taught, any light can be matched by a mixture of 3 primaries. For example, Guild had his 3 narrow bands, and Wright had slightly different ones. In this picture, the diamond shapes mark the primary wavelengths, and the curves show the match to the test light at each wavelength. The books teach us that one set of functions can be converted to another.

<on screen> **Thoughts on Matching Data**

The primary lights are not unique, and the same facts can be presented in alternate sets of graphs, an awkward situation. However,

- Wright and Guild both used red, green, and blue primaries. They are not totally arbitrary.
- The mixing of 3 primaries directly models such technologies as television. The experiment is not a strange one.
- A practical question then arises: what wavelengths work best as primaries?

<on screen> **Same Color Matches, Varying Overlap among Functions:**

To escape the changeable functions from experiments, we learn to use a fixed set of color-matching functions, namely the CIE's x-bar, y-bar, z-bar. Curiously, the CIE's functions are more overlapping than the raw data, but less overlapping than actual cone sensitivities.

<on screen> **(*x*, *y*) Diagram**

The usual teaching then steers us to the Chromaticity Diagram, a scientist's color wheel. Even in the (x, y) diagram, the issue of primary colors can be laid out. Here the + signs mark the wavelengths at which the cone sensitivities have their peaks. The colored dots mark a set of video phosphor primaries. The red cones are most sensitive to yellow light, while the red primary for color mixing is much farther in the red.

<on screen> **Which Wavelengths Act Most Strongly in Mixtures?**

MacAdam and later Thornton did calculations like this. Equal energy white at fixed power is mixed with a narrow-band light of constant power but varying wavelength. Some wavelengths act more strongly than others in perturbing the color of the white. Thornton coined the term “Prime Colors” for the red, green, and blue wavelengths that show the strongest action in these mixtures.

<on screen> **Color Matching Functions Are in Fact Relatively Stable When the Primary Wavelengths Are Changed.**

Thornton later noticed something about the functions from color matching experiments. They peak at certain preferred wavelengths, and are shifted very little by a change in the primary wavelength settings. The peaks in fact occur at the Prime Colors.

Setting the primaries themselves to the Prime Colors minimizes the optical power needed to match the test light.

A few months ago, Michael Brill discovered a new theorem. If you perturb just one primary wavelength—say the red one—then the associated function changes only by a scale factor. Its shape does not change.

<on screen> **Prime Colors Must Relate to the Overlap of Red and Green Sensitivities...**

If they are not the cone sensitivity peaks, where do the red and green Prime Colors come from? Since color vision involves comparison of the cone signals, it is logical to subtract the green cone function from the red one. The difference shows plus and minus peaks, which could be the Prime Colors. A weakness is that the scaling of the cone functions is arbitrary, and a different scaling might move the peaks.

We recall that the usual y-bar function is a sum of cone functions—with some coefficients. Normalize that function and call it ω-1. Then find a second combination of red and green that is orthogonal to ω-1, and call it ω-2.

<on screen> **One Degree of Freedom Remains, to Re-mix ω-1 and ω-2, But Keep the Mixtures Orthogonal.**

One arbitrary element remains. That is that ω-1 and ω-2 can be re-mixed to create other orthogonal pairs of functions. A rotation matrix can do this, as you see.

< **read aloud** ↠ > **Now Make a Parametric Plot, ω-2 vs ω-1.**
Bingo, the Shape is Invariant.

The graph on the right, ω-2 vs ω-1, is rotating about the origin. Each wavelength maps to a vector from the origin and the relationships among those vectors do not change. <Pause to enjoy synchronized animated graphs.>

<on screen> **Vectorial Sensitivity to Wavelength.**

So, we can go back to the earlier setup. ω-1is a function proportional to the familiar y-bar. ω-2 is a red-versus-green sensitivity. Together, they map monochromatic lights into vectors (ω_1, ω_2). The spectrum locus is the eye's vectorial sensitivity to color. It is **not** a boundary. The diagram shows a vector addition, in which one unit of yellow-green and one unit of orange add to 1.86 units of yellow.

Red circles mark the wavelengths where vector length has a local maximum. Those are the prime colors. They act strongly in mixtures.

<on screen> **Stalking Prime Colors in the 2 Dimensions of Red and Green.**

Very quickly, the idea was to make a red-green opponent-color function. We did so by choosing a red minus green function that is orthogonal to y-bar.

If we extend this method to include blue cones, then the spectrum locus in 3 dimensions is what Jozef Cohen called The Locus of Unit Monochromats. It is the eye's Vectorial Sensitivity to Wavelength. Cohen used different steps to reach this point.

<on screen> **Vectorial Sensitivity to Wavelength, Now in 3 Dimensions.**

Demonstrate VRML graph *after a few seconds!*

On the left, you see achromatic and red-green functions remain, plus a sort of blue minus yellow, which is also normalized, and orthogonal to the first two functions. Combining the 3 functions into a 3-dimensional graph gives what Jozef Cohen called the "Locus of Unit Monochromats." Cohen used different math to reach this point.

Now in this virtual reality graph, the arrows denote vectorial sensitivity at certain wavelengths. But in fact, all points along the edge of the colored surface denote similar vectors.

<on screen> **Features of the Orthonormal System**

The orthonormal scheme has many appealing features:

1. The axes have intuitive meanings: Achromatic, Red-Green, and Blue-Yellow.
2. The first two functions, $\omega 1(\lambda)$ and $\omega 2(\lambda)$, combine red and green cones only.
3. $\omega 1(\lambda)$ is a multiple of the familiar y-bar.
4. $\omega 1(\lambda)$, $\omega 2(\lambda)$ and $\omega 3(\lambda)$ combine to give the eye's vectorial sensitivity to color.
5. The functions are easily calculated, or get them from this link.
6. A light's tristimulus vector has the same magnitude as the light's "fundamental metamer."
7. Vector amplitude is non-arbitrary and has the units of the stimulus, such as radiance units.
8. The mystery is now gone from Prime Colors! They are the red, green, and blue that map to the longest vectors. They are the wavelengths that act most strongly in mixtures, exactly as Thornton said.
9. We don't usually learn to graph tristimulus vectors, or compute their magnitudes. Even graphs of the vector (X Y Z) would give some insight, but graphs and magnitudes mean more here.
10. Algebraic benefit: orthonormal functions simplify derivations and formulas.
11. Beginning students can be told flatly "This is the eye's vectorial sensitivity to color." Details can follow as needed.

<on screen> **Applications**

Do not show the VRML for multiprimary

On the left, you see a quick analysis of a multi-primary technology, an 8-color laser for light shows. For greatest benefit, the laser lines should map to similar vector lengths, but diverse directions. Arrows are drawn from the origin, and then scaled-down arrows are added to give the laser's total color.

Show VRML for Color Matching Functions

Color matching functions map to vectors, as seen here. The orthonormal color matching functions map to the axes themselves. y-bar plots to the $\omega 1$ axis, since the functions are proportional. Direction cosines between cmfs are preserved in the relationship of 3-vectors here.

<on screen> **Color Rendering by Light Sources**

Allow a few seconds for seeing 2 graphs, then show VRML.

To understand color rendering by lights, we can compare 2 lights that have the same tristimulus vector, a daylight phase and a mercury vapor light. The SPDs are chopped up into wavelength bands, a vector is computed for each band, then the arrows are chained in normal vector addition. The light of poor color rendering takes a shortcut to white, using little of the reddish and greenish components that red and green objects could reflect or absorb. The arrow chains preserve nearly all the information in the SPDs. Opinions and hidden assumptions play no role.

<on screen> **Oh, Yes, Calculating Tristimulus Vectors.**

$$\langle \omega_1 | L \rangle = \sum_{\lambda=360}^{830} \omega_1(\lambda) L(\lambda) = \int_{360}^{830} \omega_1(\lambda) L(\lambda) d\lambda$$

etc., 2 more eqs.

I didn't actually say how to find the vectors. The new calculation of a tristimulus vector has the same steps as the related calculation in the CIE scheme. But, the new result has tremendous benefits.

<on screen> **Issues Demystified:**

1. What wavelengths make the best primaries in a color mixing experiment?
2. Prime colors---wavelengths that act most strongly in mixtures.
3. Television phosphors.
4. Color rendering by lights. This important real-life issue is usually handled through hidden assumptions and arbitrary dogma.
5. Balance of primaries for multi-primary applications.
6. Camera sensitivities, lighting for camera systems, etc. Examples have used the 2° observer, but they generalize immediately.

<on screen> **Special Credit**

Bill Thornton and the late Jozef Cohen get major credit for the concepts that I've used. Mike Brill contributed many discussions. Cornsweet's book is a source and good background material. MacAdam and later Buchsbaum mentioned orthogonal cmf's, but MacAdam gets a demerit for disparaging Joe Cohen's work.

Background

My recent color rendering articles are a source for ideas, equations, and references. Today's presentation and much more are on my web site:

http://www.jimworthey.com .

Thank you.