Moral Hazard, Mergers, and Market Power

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Abstract

Most analysis of market power assumes that managers are perfect agents for shareholders. This paper relaxes that assumption. When managers of a multi-product firm exert unobservable effort to improve product quality, price coordination incentives tradeoff with effort incentives. This makes some intra-firm price competition optimal, explaining why many multi-product firms allow for competition between divisions. When there are effort spillovers, the optimal amount of price competition can be as great as when the products are under separate ownership. Even with some profit-sharing, intra-firm price competition can reduce quality-adjusted price, which has important implications for antitrust policy.
1 Introduction

Despite the fact that the last few decades of research on the theory of the firm has shown that managers will often not act as a perfect agent for the firm’s shareholders, most analyses of mergers and market power continue to assume that any multi-product firm will choose its decision variables to maximize the joint profits of the products within the firm. This paper is one of the first to relax (at least in some respects) that assumption and examine the impact on the pricing, innovation, and incentive strategies of multi-product firms.\footnote{Alternatively, one could view this paper as a model of the optimal cartel contract.} This paper shows that there are circumstances where, in order to create incentives to alleviate a managerial moral hazard problem, a multi-product firm will create substantial competition between the two products. Thus, this paper provides a possible explanation for why many multi-product firms, such as Time-Warner, before its merger with AOL, operate their different divisions as independent profit centers even though this generates competition between the divisions and reduces the firm’s ability to exercise the market power associated with owning more than one product in the same market. In fact, recent empirical evidence shows that merged firms are frequently organized in a multisubsidiary form, where the old firms are kept as fully functioning affiliates with the discretion to make independent decisions, and target management is often retained (Prechel, Boies, and Woods 1999; Zey and Swenson 1999). Such independence is not optimal in traditional market power theory, but the model in this paper demonstrates that in some circumstances it is an optimal because it provides better incentives for managers to exert unobservable effort that improves the profitability of their product.

The paper not only provides an explanation for the organization and managerial compensation schemes of many multi-product firms, it also has important implications for antitrust policy since, in differentiated products industries, mergers often result in multi-product firms. I show that where managerial moral hazard is important and centralized price setting is not feasible, some mergers may result in little or no reduction in price competition. This can happen even in many situations where standard merger policy would view the merger as almost certainly anti-competitive. In fact, I demonstrate that a merger that would increase quality-adjusted prices if there were no moral hazard problem (or centralized price setting was feasible) can actually decrease quality-adjusted prices when the firm must provide profit-based incentives to alleviate moral hazard. I also provide comparative statics results that indicate when this type of moral hazard problem is likely to have greater or lesser effect on the degree of price competition after a merger. Finally, I
discuss the implications this managerial moral hazard problem has on the profitability of mergers under Bertrand and Cournot competition: it tends to make mergers in Bertrand markets less profitable (though, they are still profitable) and may make some mergers in Cournot markets profitable when they would have been otherwise unprofitable.

This paper is concerned with the following type of situation. There are two owner managed firms, A and B, that make software for some industry. While these products both meet similar needs for similar groups of firms, they are not identical. Now, imagine that A and B merge. Since these products are far from perfect (they are software products, after all), and each manager knows her product far better than does the other manager, each must remain as part of the combined firm to improve her product. Just because they are part of one firm, however, does not mean they will now only act in their joint, as opposed to their individual, interest. While the managers might wish to set prices jointly, actual net prices may not be contractible. Software is constantly evolving, hence it may not be possible to describe the good to be sold in advance. In addition, a manager could circumvent a pre-set price by selling a product with more features or by offering more extensive installation help or other services to customers than was originally contemplated. All these factors will often prevent managers from setting price collectively. As a result, each manager will often have, at least de facto, authority to set the price for her product (and, as mentioned above, this is consistent with what actually happens in many mergers).

Of course, since both managers now jointly own the two products, they will want to reduce or eliminate price competition between them. One way to accomplish this would be to for them to agree to equally share the profits from the sales of both products (I call this complete profit-sharing). The division of profits, however, not only affects pricing incentives, but it also affects a manager’s incentive to improve her product. If product quality is unverifiable, then product profit is the only available instrument for inducing the managers to engage in costly, product-improving effort. With complete profit-sharing, each manager will bear all of the cost of product improvement (the effort required to fix bugs or add new features to the software) but receive only half the benefit (the increased profit earned by the two products together when one of them is better). To improve effort or innovation incentives, incomplete profit-sharing is optimal, even though this creates some price competition between the two products.

For the same reason that the managers do not want to completely sacrifice innovation incentives to eliminate price competition, one might also think that they will not want to completely sacrifice pricing coordination to maximize innovation incentives. If innovative effort is of purely private
value (that is, it has no effect on the quality of the other product), then this intuition is correct. If there are innovation spillovers (when one manager figures something out about how to fix a bug or add a feature it makes it easier for the other manager make her product better), however, then this is not necessarily the case.\footnote{In fact, it can be optimal (at least if one ignores the possibility of sabotage) for manager A to give manager B an added bonus based on the profit generated by product B and vice versa.} Spillovers can create a positive externality associated with product improving effort, even if each manager keeps the entire profit from her product.\footnote{I say ”can be” because even with though spillovers create a positive externality, there is also a negative externality associated with business stealing that could make the net external effect negative.} If the private value from this effort is enough greater than the external value, effort incentives will be stronger the less profit-sharing there is.

Determining the optimal level of profit-sharing between the two products, of course, is critical to assessing the competitive impact of the merger of A and B. If innovation incentives are likely to be very important relative to the effect of intra-firm price competition, then one might expect very little profit-sharing, and thus very little harmful impact on consumers. In fact, if there are spillover effects (that can only be realized when A and B are part of the same firm), the merger might even benefit consumers. This argument seems very similar to the claim often made by merging companies that improved efficiency from the merger will result in lower prices. The important difference, however, is that when product improving effort is non-contractible, spillover effects not only have the obvious direct effect on product quality but also have an indirect effect on prices through their effect on the optimal amount of profit-sharing. By making effort incentives relatively more important, spillover effects (so long as they are not too large) can cause the merged firm to create substantial price competition between the two products (despite the common ownership), leading to lower prices.

This paper is related to the extensive literature on the relationship between managerial incentives schemes and product market competition. Fershtman and Judd (1987) and Sklivas (1987) showed that a firm may profit from giving its manager incentives not to maximize profits in order to soften the behavior of competitors. More closely related are the papers that extend this analysis to multiproduct firms. For example, Vickers (1985) showed that inducing competition between divisions can be profitable because of its effect on the behavior of competitors. Fauli-Oller and Giralt (1995) and Barcena-Ruiz and Espinosa (1999) explore in greater detail the relationship between the types of incentives given to managers of multi-product firms and product market competition. These papers all assume that firms can commit to these incentive schemes. As Katz
(1991) has shown, if such commitments are not credible (because, for example, contracts can be secretly renegotiated), then these schemes for inducing competitors to behave more softly will not be effective.

Huck, Konrad, and Müller (2001) analyze mergers between identical Cournot competitors. They assume that the merged firms are kept as separate entities and continue to maximize the profits of the individual entity. Because the merger gives the two entities more information about the production of the entity within the same firm, however, it allows one firm to act as a Stackelberg leader with respect to the other, increasing the combined profits of the firm and total quantity while reducing price. Thus, keeping the entities separate makes a horizontal merger profitable when it otherwise would not be. In none of these papers, however, is unobservable managerial effort an issue. So, in some sense, my model performs the opposite analysis of these papers. It looks at the effect of the need to motivate managers on product market competition rather than the effect of product market competition on how managers are compensated.

The literature on research joint ventures (e.g., Kamien, Muller, and Zang 1992; Ziss 1994; Vonortas 1994; Cabral 2000) also shares some similarities with this paper. These papers, however, typically assume that the joint venture can contractually fix the level of research and development (R&D) done by participating firms. The one exception is Cabral (2000). He shows that R&D cooperation can be sustained even when R&D effort is unobservable. Unlike this paper, however, his model assumes complete R&D spillovers between the participating firms and completely rules out any degree of profit-sharing. So, his paper is not aimed at examining the interaction between effort incentives and profit-sharing for varying degrees of spillovers.

The literature on revenue sharing in partnerships (Gaynor 1989; Gaynor 1990) is related to this paper, though it assumes price can be set collectively. Fulghieri and Hodrick’s (1997) paper on the interaction of synergies and agency conflicts for multi-divisional firms also discusses similar issues, though its focus is on conflicts between managers who are paid a fixed wage and a board of directors that acts in the best interests of the shareholders.

In some ways, this paper is an application of Holmstrom’s (1982) work on moral hazard in teams. In that paper, individual signals of effort were unavailable, whereas here they are available but distortionary. Holmstrom’s insight that using only a joint signal will lead to suboptimal effort incentives is what drives the main result in this paper that it can pay to use individual signals of effort even when doing so reduces the total surplus available for any given effort level.

The plan of the paper is as follows. The next section outlines the general model. Section
3 examines the linear demand case with 3.1 discussing optimal profit-sharing, 3.2 discusses price effects, and 3.3 discusses how moral hazard changes the analysis of mergers. Section 4 discusses how these results might affect merger profitability when there are additional competitors. Section 5 concludes. All proofs are in the Appendix.

2 Model

There are two managers, each of which has specialized expertise over one product. These two products compete with each other in the sense that the price and quality of one product affects the demand for the other product. Each manager can exert unobservable effort to improve the quality of her product. I assume the demand function for each product is symmetrical, thus the demand for product $i$ (where the other product is $-i$) can be written as follows:

$$q^i = q(p^i, p^{-i}, e^i, e^{-i})$$

(1)

Here $e^i$ represents the quality improving effort made by manager $i$. I will assume that one’s own price and effort effect one’s demand at least as much as the price and effort of the other manager, that is, $q_1 + q_2 \leq 0, q_3 + q_4 \geq 0$. Since I assume that each product is produced with a constant marginal cost, $c$, product $i$ generates the following profit:

$$\pi^i = (p^i - c)q(p^i, p^{-i}, e^i, e^{-i})$$

(2)

There are three periods in the model. In period 1, the two managers choose the fraction, $\alpha$, that manager $i$ will receive from the profit of product $i$. In period 2, manager $i$ chooses her price, $p^i$, and level of non-contractible effort, $e^i$. In period 3, effort costs and profits are realized and distributed according to the level of profit-sharing determined in period 1. Some comments about this structure are in order.

First, note that if manager $i$ receives $\alpha$ of the profit from product $i$, then I require that manager $-i$ receive the remaining $1 - \alpha$ of this profit. That is, I assume budget balance. While in many circumstances, it might be ex ante optimal to commit to give away some or all of the profits, this will never be ex post optimal. As a result, the manager’s would always renegotiate such a contract.

\[\text{\footnotesize{\textsuperscript{4}Throughout the paper, I restrict analysis to } \alpha \geq 1/2. \ \text{Unless spillover effects are very large, so that effort is decreasing in } \alpha \text{ even for } \alpha < 1/2, \ \alpha < 1/2 \text{ will never be optimal. I always restrict the magnitude of spillovers so that } \alpha \geq 1/2 \text{ is optimal.}}\]
They could contract with a third party to make this commitment binding, but then such a deal is vulnerable to collusion with that third party. Moreover, if demand is subject to exogenous shocks, contracts that offer payments to third parties if the products do not earn first-best profits will, even in equilibrium, result in wasteful payments to third parties, even if there is no risk of collusion. Alternatively, a third party could agree to give both managers all the profits from both products, generating first-best pricing and effort incentives. If the managers have limited wealth, however, they will not be able to pay someone enough to participate in such a contract. Since contracts where managers make up-front payments and receive (collectively) more than all the profits the firm earns rarely occur, I do not allow them.

I also restrict the analysis to symmetric profit-sharing contracts. This greatly simplifies the analysis. Since the model is symmetric, this is optimal so long as the products are not such close substitutes that it is optimal not to provide incentives for product improving effort to one manager.\(^5\) In period one, there is no conflict among the managers; since each manager has a symmetrical utility function, each wants to choose \(\alpha\) to maximize that function.

The assumption that neither quality-improving effort nor price can be jointly set in this period is critical. If effort were contractible, then it would clearly be optimal to set \(\alpha = 1/2\) so that neither manager would have any incentive to lower price to steal customers from the other. On the other hand, if prices could be set jointly, then \(\alpha\) would only affect a manager’s effort incentives. Both managers would agree to set prices at the joint profit-maximizing level and set \(\alpha > 1/2\) so that they would not exert too little effort.\(^6\) While I certainly do not claim that joint price setting is always impossible, there will be many situations where it is not feasible. As discussed in the introduction, if the product is constantly evolving (as the existence of product-improving effort implies), then it will be very difficult to describe the product to be sold in advance. Moreover, if each manager sells many different versions of her product at different prices (and offers services along with them), the sales of each type of product and service may not be verifiable even if total revenues are. The very fact that, as discussed above, after many mergers, the previously independent firms are kept as affiliates with the authority to make independent pricing decisions suggests that this assumption accurately depicts the situation following many, though certainly not all, mergers.\(^7\)

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\(^5\) If the products are very close substitutes, then it may be optimal for the firm to give almost all of the profit from both products to one manager. This way, price competition can be eliminated and one manager will have very strong effort incentives.

\(^6\) \(\alpha = 1\) would not necessarily be optimal since, if effort spillovers were minor, then there would be excessive incentives to exert effort so as to steal customers from the other product.

\(^7\) This also suggests that repeated interaction is often not sufficient to alleviate this problem. Even if repeated
The solution concept I use for this model is subgame perfection. I solve the model using backward induction. In period 2, each manager chooses price and effort to maximize her utility given $\alpha$. If exerting quality improving effort $e^i$ imposes a private and unobservable cost $k(e^i)$ on manager $i$, then her utility is given by:

$$u^i(\alpha) = \alpha\pi^i + (1 - \alpha)\pi^{-i} - k(e^i) \quad (3)$$

By differentiating (3), this gives the following first order conditions for $p^i$ and $e^i$:

$$\alpha(q^i + (p^i - c)q_1^i) + (1 - \alpha)(p^{-i} - c)q_2^{-i} = 0 \quad (4)$$

$$\alpha(p^i - c)q_3^i + (1 - \alpha)(p^{-i} - c)q_4^{-i} - k'(e^i) = 0 \quad (5)$$

Throughout the paper, subscripts will be used to denote partial derivatives. Thus, $q_1^i$ is the derivative of $q^i$ with respect to its first argument (which is $p^i$) and $q_2^{-i}$ is the derivative of $q^{-i}$ with respect to its second argument (which is $p^i$). In (4), the term multiplied by $\alpha$ is the standard condition for profit maximization by a one product firm. The second term is the adjustment due to the fact that manager $i$ also has a stake in the profits generated by product $-i$, whose sales are affected by the price of product $i$. Similarly, in (5), the term multiplied by $\alpha$ reflects the effect of increasing effort on the demand for a manager’s own product and the term multiplied by $1 - \alpha$ reflects the effect of effort on the demand for the other manager’s product.

These four first order conditions (one each for $i = 1, 2$) implicitly determine $p^i$ and $e^i$. Because of the symmetry in the model, one can write $p^i = p(\alpha)$ and $e^i = e(\alpha)$ for $i = 1, 2$. The optimal degree of profit-sharing in period 1 can then be determined by differentiating (3) with respect to $\alpha$. Doing so, using the pricing and effort first order conditions and rearranging terms, gives the following marginal utility of $\alpha$ for each manager:

$$2(p - c)[p'(\frac{2\alpha - 1}{\alpha})q_2 + e'(\alpha q_4 + (1 - \alpha)q_3)] \quad (6)$$

If arbitrary $\alpha$ is feasible, then (6) must equal zero at the optimal $\alpha$. If, because of limited liability or concerns about sabotage, $\alpha \leq 1$, then no profit-sharing could be optimal even though (6) is

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Footnotes:
1. Interaction did allow the firm to support higher prices, this cannot be based on verifiable agreements. Thus, the fact that the two products are under common ownership would not affect the ability of repeated interaction to generate collusive prices.
2. The second order conditions for maximization will hold so long as $q_{11} < 0$, $q_{22} < -q_{11}$ and $k$ is sufficiently convex.
positive at $\alpha = 1$.

Increasing $\alpha$ affects a manager’s utility through its effects on price and quality improving effort. Of course, a manager knows that she will set her price and effort level to maximize her utility, so her concern is how $\alpha$ affects the other manager’s pricing and effort incentives. The first term represents the price effect. If $p' < 0$ (more profit-sharing leads to higher prices)$^9$, then this effect is negative so long as each manager retains more than half of the profit from her own product, $\alpha > 1/2$, and the products are substitutes, $q_2 > 0$. The second term represents the effort effect. If $e' > 0$ (less profit-sharing leads to more effort), then this effect is negative whenever an equal increase in quality for both products increases the demand for each product. Notice that when when $\alpha = 1/2$, there is complete profit-sharing, increasing $\alpha$ will increase utility as long as $e'(1/2) > 0$. ($\alpha > 1/2$ will be globally optimal whenever $p'(\alpha) < 0$ and $e'(\alpha) > 0$ for all $\alpha \leq 1/2$.) That is, if $e'(1/2) > 0$, then the managers want less than complete profit-sharing.

Intuitively, incomplete profit-sharing is not surprising, it follows from the envelope theorem. With complete profit-sharing, the pricing distortion from increasing $\alpha$ (less profit-sharing) is of second order while the effort effects are of first order. Since the incentive for product improving effort is sub-optimal when a manager bears all the cost but only gets half the benefit, increasing $\alpha$ above $1/2$ makes the managers better off.

Unfortunately, with a fully general demand function the conditions necessary to guarantee that effort will increase when moving away from complete profit-sharing (and that prices will decrease) are quite complicated and not very illuminating. While the direct affect of increasing $\alpha$ is positive (keeping more of one’s profits increases the incentive to make the good more profitable) there is also an indirect effect. Increasing $\alpha$ causes the other manager to cut price, reducing the first manager’s sales which reduces the incentive to increase quality. This price reduction can also affect the magnitude of the effect of effort on demand.

If demand is only a function of the quality-adjusted price for each product, however, then it is straightforward to show that decreasing profit-sharing from the complete profit-sharing level will increase effort. Because the units of quality are not important, this does not restrict how quality affects demand. It does, however, restrict the second order effects. That is, it restricts how the effect of quality on demand changes as prices or quality changes. Such restrictions are

\footnote{Because effort and price decisions interact, the conditions necessary to guarantee that $p'(\alpha) < 0$ are quite complicated and unintuitive. Notice that if $e'(\alpha) > 0$, then increasing $\alpha$ increases the quality of the product, which induces higher prices, counter-acting the reduced incentive to collude. If demand is only a function of the quality-adjusted prices of both products, however, then one can show that $p'(\alpha) < 0$ whenever $k$ is sufficiently convex.}
not necessary, but they are sufficient for the following proposition to hold.

**Proposition 1** If demand can be written as 
\[ q_i = q(p_i - (e^i + \gamma e^{-i}), p^{-i} - (e^{-i} + \gamma e^i)) \] (with \(0 \leq \gamma < 1\)), then complete profit-sharing (\(\alpha = 1/2\)) is never optimal as long as 
\[ q_{11} + 2q_{12} + q_{22} \leq 0, \] and spillovers not too large (\(\gamma \leq \frac{q_{11} + q_{22}}{q_{11} - 3q_{12}}\)) and \(k\) is sufficiently convex (\(k(e) \geq -(1 + \gamma^2)(q_1 + q_2)/4\)).

This proposition shows that even when both products are under common ownership, there will still be some price competition between them. The need to motivate managers to engage in quality-improving effort prevents the merger from achieving the perfectly collusive outcome.

### 3 Linear Demand Case

In order to more fully explore the effect of unobservable effort on the degree of competition between managers in the same firm, I fix a demand and effort cost function. Using a linear demand specification (or, indeed, any specific demand specification) is restrictive. With a fully general demand curve, however, one can only determine whether there will be intra-firm price competition, but one cannot say how important it will be. The following analysis demonstrates that intra-firm price competition can be substantial. Because functional forms are specified, the results are only suggestive of what is possible. Nonetheless, they demonstrate that merger analysis cannot ignore the possibility that moral hazard may significantly impact the ability of a multi-product firm to exercise market power.

In this section, I assume that the effort cost function is given by effort cost function of 
\[ k(e) = ke^2 \] and that demand is a linear function of quality-adjusted price:
\[
q^i = a - b_1(p^i - e^i - \gamma e^{-i}) + b_2((p^{-i} - e^{-i} - \gamma e^i) - (p^i - e^i - \gamma e^{-i}))
\] (7)

When \(b_2 = 0\), demand for one product is unaffected by the other product. When \(b_2 = \infty\), the two products are perfect substitutes. Quality enhancing effort by the manager is measured in units of quality improvement. Moreover, I allow for effort spillovers. When manager \(i\) exerts effort \(e^i\) this not only improves her product by \(e^i\), it also improves product \(-i\) by \(\gamma e^i\). The existence of such spillovers is a common, pro-competitive, justification for the merger of two competing products. When these spillovers result from unobservable effort, however, they may indirectly affect the quality-adjusted price paid by consumers through an effect on the optimal degree of intra-firm price competition, in addition to having a direct effect on product quality.
Notice that I can also write the effort cost function as effort cost function of \( k(e) = \tilde{k} b_1 e^2 \) and the demand function as follows:

\[
q_i = a - b_1[(p^i - e^i - \gamma e^{-i}) + \tilde{b}_2((p^{-i} - e^{-i} - \gamma e^i) - (p^i - e^i - \gamma e^{-i}))]
\]  

(8)

Doing this parameter transformation (\( b_2 = \tilde{b}_2 b_1 \) and \( k = \tilde{k} b_1 \)) will be helpful below. When demand is given by (8), the first order conditions for \( p^i \) and \( e^i \) \((i = 1, 2)\) become the following:

\[
\alpha(a - b_1(2p^i - e^i - \gamma e^{-i} - c)) + \tilde{b}_2 b_1(p^{-i} - 2ap^i + \alpha(1 - \gamma)(e^i - e^{-i}) + (2\alpha - 1)c) = 0
\]  

(9)

\[
\alpha(p^i - c)(1 + \tilde{b}_2(1 - \gamma)) + (1 - \alpha)(p^{-i} - c)(\gamma - \tilde{b}_2(1 - \gamma)) - 2\tilde{k}e^i = 0
\]  

(10)

Solving these four first order conditions for \( p^i \) and \( e^i \) gives the following:

\[
p^i = \frac{2\alpha a \tilde{k} + b_1 \{\alpha[2\tilde{k} - (1 + \gamma)(\alpha(1 - \gamma) + \gamma)] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma)^2)\}}{b_1 \{\alpha[4\tilde{k} - (1 + \gamma)(\alpha(1 - \gamma) + \gamma)] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma)^2)\}}
\]  

(11)

\[
e^i = \frac{\alpha(a - b_1 c)\{\gamma + (1 - \gamma)(\alpha + (2\alpha - 1)\tilde{b}_2)\}}{b_1 \{\alpha[4\tilde{k} - (1 + \gamma)(\alpha(1 - \gamma) + \gamma)] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma)^2)\}}
\]  

(12)

Before determining the optimal level of profit-sharing, it is instructive to examine how price and effort vary with \( \alpha \). Differentiating the right hand side of (12) with respect to \( \alpha \) gives the effect of \( \alpha \) on effort or quality:

\[
\frac{2(a - b_1 c)\tilde{k}\{(1 - \gamma)(2\alpha^2 + 2\alpha(3\alpha - 1)\tilde{b}_2 + \tilde{b}_2^2(2\alpha - 1)^2) - \tilde{b}_2\gamma\}}{b_1^2\{\alpha[4\tilde{k} - (1 + \gamma)(\alpha(1 - \gamma) + \gamma)] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma)^2)\}^2}
\]  

(13)

This is positive if and only if the term in curly braces in the numerator is positive. This term is increasing in \( \alpha \). Hence, to obtain a sufficient condition for increasing \( \alpha \) to increase effort, I evaluate this term at \( \alpha = 1/2 \). The term is then positive if and only if:

\[
\gamma \leq \frac{1 + \tilde{b}_2}{1 + 3\tilde{b}_2}
\]  

(14)

As long as spillovers are not too large, increasing the share of the profit of her own product that a manager retains increases her incentive to exert effort. The reason this does not hold for all \( \gamma < 1 \) (unless the products are independent) is that by inducing lower prices, increasing \( \alpha \) reduces benefit
from increasing sales through increasing quality.

Because price can increase because of increasing quality, it is more instructive to examine how quality-adjusted price, \( p^i - e^i - \gamma e^{-i} \), changes with \( \alpha \) than it is to look at the effect of \( \alpha \) on price itself. Using (11) and (12), \( \frac{d(p^i - e^i - \gamma e^{-i})}{d\alpha} \) is:

\[
- \frac{2(a - b_1 c)\tilde{k} \{(1 - \gamma^2)(\alpha + \tilde{b}_2)(2\alpha - 1)) + \tilde{b}_2(2\tilde{k} - (1 + \gamma)\gamma)}{b_1^2 \{\alpha[4\tilde{k} - (1 + \gamma)(\alpha - \gamma) + \gamma] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma^2))\}^2}
\]

(15)

Increasing the share of the profit of a manager’s product that she retains will decrease the quality-adjusted price she charges for that product whenever the term in the curly braces in the numerator is positive. With some simple algebraic manipulation, it is easy to show this is the case whenever (12) generates positive effort, effort is increasing in \( \alpha \), and the second order conditions for \( p \) and \( e \) are satisfied.

### 3.1 Optimal Profit-sharing

By using the explicit solutions for price and effort, one can get a reduced form expression for utility that can be optimized over \( \alpha \). This gives the following first order condition for \( \alpha \):

\[
0 = \frac{4(a - b_1 c)^2\tilde{k}^2}{b_1^2} 2\alpha^3(1 - \gamma)(1 - \alpha(1 - \gamma)) - \alpha\tilde{b}_2[\gamma + 3\alpha(1 - \gamma) - 2\alpha^2(6 - \alpha)(1 - \gamma) + 10\alpha^3(1 - \gamma)^2] - \tilde{b}_2^2(2\alpha - 1)[2\tilde{k} + \alpha(1 - \gamma)(1 - \alpha(7 - \gamma) + 8\alpha^2(1 - \gamma) - \gamma)] - \tilde{b}_2^2\alpha(2\alpha - 1)^3(1 - \gamma)^2 \]

\[
\{\alpha[4\tilde{k} - (1 + \gamma)(\alpha - \gamma) + \gamma] + \tilde{b}_2(2\alpha - 1)(2\tilde{k} - \alpha(1 - \gamma^2))\}^2
\]

(46)

Since the first fraction is independent of \( \alpha \), the optimal \( \alpha \) depends only on \( \tilde{b}_2 \) (the normalized measure of how close substitutes the products are), \( \tilde{k} \) (the normalized effort cost parameter), and \( \gamma \) (the magnitude of the effort spillovers). The following proposition describes how the optimal \( \alpha \) varies with these parameters.

**Proposition 2** Let demand be given by (8) and say \( \tilde{k} > 1/2 \) and \( \gamma < .3 \). The optimal \( \alpha \) is decreasing in \( \tilde{k} \) and \( \tilde{b}_2 \) and is increasing in \( \gamma \) if and only if \( \gamma < \hat{\gamma} \).

Not surprisingly, as the products become closer substitutes the more important it is to reduce the incentive for intra-firm price competition. Similarly, as the cost of effort increases the level

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11The second order condition is satisfied at the first order condition whenever \( \alpha, \tilde{k} > 1/2 \) and \( \gamma < 0.3 \).
of product improving effort falls. When this effort level is low, the impact of distorting effort incentives is also low. Even when \( \alpha \) is chosen to maximize effort incentives, there will still be very little effort if the cost is high enough. As a result, it is not worth inducing much price competition to get a very small improvement in effort incentives (this is due to the linear impact of effort on demand).

The effect of product improving spillovers is more complicated. On the one hand, spillovers create a positive externality for product improving effort. Thus, when spillovers are larger, it is more important to create greater effort incentives, making a larger \( \alpha \) optimal. On the other hand, the larger are spillovers, the less effective increasing \( \alpha \) is on improving effort incentives. Thus, when spillovers get too big, the negative effect (from more profit-sharing) on effort is small relative to the positive effect from reducing price competition. When the products are very close substitutes (\( \tilde{b}_2 \) is large), \( \tilde{\gamma} \) can even be negative, so that the optimal \( \alpha \) is always decreasing in \( \gamma \). On the other hand, if the products are very close substitutes, then \( \tilde{\gamma} > .3 \).

The restrictions on the magnitude of spillovers and the effort cost parameter ensure that the second order condition for \( \alpha \) is satisfied. When the second order condition is not satisfied, one can still determine the optimal \( \alpha \) via numerical methods. In these cases that I have examined, the results from Proposition 2 continue to hold even when the second order condition for \( \alpha \) is not satisfied.

I now turn to the question of how important is managerial moral hazard in generating intra-firm price competition. The next proposition establishes when managerial moral hazard will cause a multi-product firm to not share profits at all and when there will always be some profit-sharing.

**Proposition 3** Let demand be given by (8) and say \( \tilde{k} > 1/2 \). (A) If \( \gamma = 0 \), there are no effort spillovers, then the optimal \( \alpha \) is strictly less than unity. (B) If \( \tilde{b}_2 < .948 \), then there exists a \( \tilde{k} > 1/2 \) and a \( \gamma < .3 \) such that no profit-sharing is optimal.

If there are no effort spillovers, then if each manager retains all the profit from her product, there is a business stealing externality associated with product improving effort. As a result, each manager has too great an incentive to improve her product. Creating some profit-sharing will then both improve effort incentives and pricing incentives. That is, without spillovers, there will always be less competition when the two products are under common ownership than when they are under separate ownership. Moral hazard guarantees that the reduction in competition will not be as great as it would be if effort were completely contractible, but it does not eliminate it.
When there are effort spillovers, however, this is not necessarily the case, as part (B) of Proposition 3 demonstrates. If the products are not too close substitutes, then, if there are positive spillovers and effort costs are not too great, no profit-sharing can be optimal. In fact, if there are no limited liability constraints or concerns about sabotage, then \( \alpha > 1 \) could be optimal (manager \( i \) pays a bonus to manager \( -i \) that is a percentage of product \( -i \)'s profit). From Proposition 2, we know that the range of \( \tilde{k} \) and \( \gamma \) for which \( \alpha \geq 1 \) is optimal will increase the farther below \( .948 \) is \( \tilde{b}_2 \).

Given that \( \tilde{b}_2 \) can range from zero to infinity, one might think that when \( \tilde{b}_2 < .948 \) the products are close enough to being independent that a merger between these two products would never be a concern for antitrust authorities even without moral hazard. While in some cases this is true (for example, when the market is very small even when there is marginal cost pricing), in other cases such a merger could be viewed as merger to monopoly (when the market is quite large at marginal cost pricing).

According to the U.S. Department of Justice and the Federal Trade Commission’s Horizontal Merger Guidelines (1997), the two closest substitutes will be a product market by themselves if a hypothetical monopolist would find it profitable to impose a small but significant price increase. This price increase is normally taken to be five or ten percent. I now translate this standard into a standard based on \( \tilde{b}_2 \) when quality-improving effort is not important. To do so, I assume that quality is fixed and incorporate this into the constant term, \( a \), in the demand function (8). In this setting, price as a function of \( \alpha \) becomes:

\[
p^i(\alpha) = \frac{\alpha(a + b_1c) + \tilde{b}_2(2\alpha - 1)b_1c}{b_1(2\alpha + b_2(2\alpha - 1))} \tag{17}
\]

Thus, the optimal price increase from a merger is just \( p^i(\alpha = 1/2) - p^i(\alpha = 1) \), or:

\[
\frac{\tilde{b}_2(a - b_1c)}{2(a + b_1c(1 + \tilde{b}_2))} \tag{18}
\]

A price increase of at least \( x \) percent is optimal if and only if:

\[
\tilde{b}_2 \geq \frac{2x(a + b_1c)}{a - b_1c(1 + 2x)} \tag{19}
\]

It is important to note that this actually overstates the \( \tilde{b}_2 \) necessary for these two products to constitute a market by themselves. This \( \tilde{b}_2 \) insures that an \( x \) percent price increase is optimal.
while the merger guidelines require only that this price increase is profitable. Even if a smaller price increase is optimal, a larger one could still be profitable. This $\tilde{b}_2$ would also overstate the true critical value if there are other substitute products that might respond to a price increase by increasing their prices (prices are strategic complements). Incorporating only two firms in the demand function implicitly holds all other prices fixed, ignoring the strategic benefit from raising prices.

By setting in $x = .1$ in (19), one can see that a ten percent price increase will be optimal whenever $\tilde{b}_2 \geq .948$ as long as $b_1c \leq .559a$ and a five percent price increase will be optimal whenever $\tilde{b}_2 \geq .948$ as long as $b_1c \leq .742a$. In general, in industries with low marginal costs (such as software) or large demand, then United States antitrust authorities could be greatly concerned about a merger where $\tilde{b}_2$ is small enough that the existence of managerial moral hazard would mean that this merger would not reduce price competition at all. That is, there are many situations where non-contractible effort should dramatically alter optimal antitrust enforcement.

Figures 1 and 2 give some indication of how large the optimal $\alpha$ is for various parameter values. These figures all allow $\alpha > 1$. Where limited liability or sabotage concerns make $\alpha > 1$ infeasible, then $\alpha = 1$ is optimal whenever the figures show $\alpha > 1$. Because the figures use numerical analysis, they include some parameter values which I excluded in the above proposition to ensure that the second order condition for $\alpha$ holds. For example, Figure 1 shows that when $\tilde{b}_2 = 1$, for $\gamma > .3$ the optimal $\alpha$ can exceed one, though we know from Proposition 3 that when $\gamma < .3$, the optimal $\alpha$ can only exceed one for $\tilde{b}_2 < .948$.

It is worth noting that even when there are no effort spillovers, there is still some incentive for intra-firm price competition. As Proposition 2 indicates, when effort costs and/or cross-price elasticity are smaller, the optimal $\alpha$ is larger for any spillover magnitude. On the other hand, for larger effort costs and/or cross-price elasticity, the optimal degree of intra-firm price competition will decline. Moreover, as Figure 2 shows, it is in such situations that the non-monotonic effect of spillovers comes into play even at lower levels of spillovers.

### 3.2 Price Effects

While the no profit-sharing is sometime optimal, this section shows that it is not necessary for quality-adjusted prices to fall after a merger. If the amount of profit sharing is sufficiently small, the existence of quality-improving spillovers from the merger can lead to lower quality-adjusted price even though there is some profit sharing. Because I assume that effort spillovers occur
Figure 1:

Figure 2:
only when the products are under common ownership, the quality-adjusted price for product \( i \) is:

\[
P_i^i = p_i^i - e_i^i - I_i \gamma e_i^i
\]

where \( I \) is an indicator function that takes the value one if the two products are in one firm and zero otherwise. Computing the pre-merger prices is straightforward since the pre-merger prices and efforts are given by (11) and (12) with \( \alpha = 1 \) (no profit-sharing) and \( \gamma = 0 \) (no spillovers). Computing the post-merger prices and efforts requires solving (16) to get the optimal \( \alpha \) for the given parameter values and then using this value along with the other parameter values in (11) and (12). Since (16) is a quartic equation, the analytic solutions are extremely cumbersome and different one’s are valid for different parameter values. As a result, I solve for \( \alpha \) using equation (16) numerically for various parameter values and the compare the pre- and post-merger quality-adjusted prices for those parameter values. Fortunately, since the optimal \( \alpha \) is independent of \( a, b_1, \) and \( c \), it turns out that the direction (though not the magnitude) of the post-merger quality-adjusted price change is also independent of \( a, b_1, \) and \( c \). The following table gives the minimum degree of spillovers necessary for a merger to result in a decrease in quality-adjusted price (\( \Delta P_i^i < 0 \)) for various values of \( \tilde{k} \) and \( \tilde{b}_2 \). Of course, as Proposition 2 suggests, there is also a maximum value since for very large degrees of spillovers the optimal degree of profit-sharing increases dramatically. This maximum is not shown, but it is larger where the minimum value, given in the table, is smaller.

### Table 1

<table>
<thead>
<tr>
<th>( \tilde{k} )</th>
<th>( \tilde{b}_2 = \frac{1}{4} )</th>
<th>( \tilde{b}_2 = \frac{1}{2} )</th>
<th>( \tilde{b}_2 = \frac{3}{4} )</th>
<th>( \tilde{b}_2 = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{k} = \frac{1}{2} )</td>
<td>.067</td>
<td>.132</td>
<td>.192</td>
<td>.249</td>
</tr>
<tr>
<td>( \tilde{k} = \frac{3}{4} )</td>
<td>.074</td>
<td>.154</td>
<td>.233</td>
<td>.311</td>
</tr>
<tr>
<td>( \tilde{k} = 1 )</td>
<td>.082</td>
<td>.179</td>
<td>.282</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = \frac{5}{4} )</td>
<td>.090</td>
<td>.206</td>
<td>.350</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = 2 )</td>
<td>.098</td>
<td>.237</td>
<td>DNE</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = \frac{3}{2} )</td>
<td>.116</td>
<td>.321</td>
<td>DNE</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = 3 )</td>
<td>.156</td>
<td>DNE</td>
<td>DNE</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = 4 )</td>
<td>.204</td>
<td>DNE</td>
<td>DNE</td>
<td>DNE</td>
</tr>
<tr>
<td>( \tilde{k} = 6 )</td>
<td>.364</td>
<td>DNE</td>
<td>DNE</td>
<td>DNE</td>
</tr>
</tbody>
</table>

When \( \tilde{b}_2 = 1/4 \), despite the fact that standard antitrust analysis would, in many markets, find this merger to be a merger to monopoly, even quite small spillovers will result in a decrease in quality-adjusted prices provided the normalized effort cost parameter is not too great. When intra-firm competition is necessary to motivate managerial effort, standard merger analysis may be far too

---

12 The values in this table were generated by numerically solving for the value of \( \gamma \) for which \( \Delta P_i^i = 0 \) using Newton’s Method. This method cannot be used to determine the maximum value of \( \gamma \) for which \( \Delta P_i^i = 0 \) since at this maximum there is a discrete jump from \( \Delta P_i^i < 0 \) to \( \Delta P_i^i > 0 \). DNE (which stands for ”does not exist”) means that there is no value of \( \gamma \) for which the merger results in a decrease in the quality-adjusted price.
strict. Of course, whether or not this is the case (for any given degree of product substitutability) will depend upon estimates of the degree of effort spillovers and the cost of quality improving effort. In order to accurately assess the competitive impact of a merger (where managers must be given profit-based incentives to induce product-specific, quality-enhancing effort), one must estimate not only the demand parameters and the synergies from the combination but also the innovation cost parameters. Of course, if estimating the demand parameters shows that the products are very close substitutes, then (as Table 1 indicates) it is very unlikely that the merger will result in a quality-adjusted price decrease no matter what the value of the other parameters.

3.3 Moral Hazard Effects

The last section shows how spillovers affect whether a merger results in an increase or decrease in quality-adjusted price. Of course, positive spillover effects are often cited as a reason why an otherwise anti-competitive merger might be pro-competitive. Nevertheless, the effects of spillovers in this model are substantially different than they are in the standard model where moral hazard is not an issue. First, observe that in such a model it is optimal for there to be complete profit-sharing ($\alpha = 1/2$) so that neither manager has an incentive to steal business from the other. Since effort is contractible, it will be determined before price. I solve the price first order conditions, (9), for prices given efforts with $\alpha = 1/2$. Given these prices, the optimal effort level is given by the $e_i$ that maximizes $u'(1/2) + u^{-i}(1/2)$. Setting this first order condition equal to zero, imposing symmetry, and solving for $e_i$ gives the following:

$$e_{i,NMH} = \frac{(a - (1 - \hat{b}_2)b_1c)(1 + \gamma)}{4k - (1 + \gamma)^2(1 - \hat{b}_2)b_1}$$

(20)

The formula for post-merger quality-adjusted prices with no moral hazard is $p_{i,NMH} - (1 + \gamma)e_{i,NMH}$ where $p_{i,NMH}$ is determined as above with efforts given by (20). Pre-merger quality-adjusted prices are determined as in the previous sub-section since they are unaffected by the observability of effort since when each product is owned separately there is no one with whom to contract. The quality-adjusted price change in the moral hazard case is determined as in the last sub-section.

Unlike the moral hazard case, in the no moral hazard case, the merger-induced change in quality-adjusted price is monotonically decreasing in the magnitude of the spillovers. This is not surprising given that larger spillovers increase the benefits of quality improving effort when there
is a merger but have no effect otherwise. Since it is not optimal to increase price one for one with an improvement in quality, greater improvements in quality (as a result of the merger) result in smaller increases or greater reductions in the quality-adjusted price.

Figure 3 compares the minimum magnitude of effort spillovers necessary for a merger to result in a decrease in quality-adjusted prices as a function of the effort cost parameter in the moral hazard and no moral hazard cases.

While the figure considers the case of $\tilde{b}_2 = 1/4$, the picture is qualitatively similar for larger values of $\tilde{b}_2$. The main differences are that both lines are shifted up and, as is clear from Table 1, they end at $\tilde{k} < 5$. Figure 3 demonstrates that there is a substantial region of spillover magnitudes and effort costs where a merger decreases quality-adjusted price if product improving effort is non-contractible but increases it if this effort is contractible. Since we also know (though it is not depicted in the figure) that for very high levels of spillovers, quality-adjusted price can increase as a result of a merger when it would decrease for intermediate spillover levels, the reverse is also true. There is a region above the dashed line in Figure 3 where a merger with contractible effort decreases quality-adjusted price while the same merger where effort is not contractible increases quality-adjusted price. Thus, to accurately assess the impact of any given merger on consumer welfare one must not ignore the contractibility of product improving effort.

4 Oligopoly and The Profitability of Mergers

The above model assumes that the only competing products are the two produced by the two managers. That is, it assumes that these products are distant enough substitutes for all other
products that the prices of all other products are not affected by whether or not the two products are in one firm or two. In this section, I discuss how the results would change if these two products are in an oligopoly market and examine the effect of moral hazard on the standard results concerning the profitability of mergers.

When firms do not suffer from agency problems, it is well-known that (ignoring the possibility of entry and merger induced cost-savings) that mergers are profitable when firms compete in prices (Deneckere and Davidson 1985) but not when they compete in quantities (Salant et. al. 1983). The results derive from how other firms in the industry respond to the new incentive of the merged firm to choose its strategic variable (price or quantity) to maximize the joint profit of two, previously independent, firms. If the merging firms suffer from the moral hazard problem described above, however, I show that it is not optimal for the firm to induce its managers to choose their price (or, analogously, quantity) to maximize the joint profits of the firm. This result will carry over to the oligopoly setting since the existence of other competitors will not affect the optimal degree of profit-sharing so long the magnitude of profit-sharing is not observable or can be renegotiated (this is proven in Katz (1991)).

If there are no effort spillovers, then we know from Proposition 3 that there will be some profit-sharing. Because profit-sharing leads to higher prices, when firms compete in prices, rivals should increase their prices in response to the merger. Thus, the merger will continue to profitable. It will not be as profitable as in the standard situation both because moral hazard leads to some intra-firm price competition and because the smaller price increase will lead to smaller price increases from rivals. If there are effort spillovers leading to no profit-sharing, then the divisions in the merged firm will be in exactly the same competitive position (given product quality) they were before the merger. The merger will be profitable, however, because the quality improving spillovers increase the quality of products of the merged firm relative to those outside the merged firm. This is a merger that is profitable for the insiders, unprofitable for the outsiders, and welfare-enhancing.

Because the model assumes price, rather than quantity, competition, it is not directly applicable to Cournot games. Nonetheless, it can provide intuition as to how moral hazard might affect the

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13 If $\alpha$ is unobservable, then the managers take the price of other firms as given. The fact of the merger may change the price other firms’ charge, but this will just be like changing the intercept, $a$, in the demand curve, which does not affect the optimal $\alpha$.

14 If one allows $\alpha > 1$, some mergers could lead to price cutting by rivals. I have found no cases, however, where this effect was sufficient to cause such a merger to be unprofitable. Even if one could find such a case, a firm might be able to commit to separating the managers (physically, for example) to eliminate the effort spillovers that would make $\alpha > 1$ optimal, making the merger profitable.
profitability of mergers when firms compete in quantities. Since without moral hazard, mergers in these games are unprofitable, it is likely that if there is a great deal of profit-sharing (α is near one-half), then mergers will remain unprofitable unless the spillover benefits are large enough to compensate for the strategic disadvantage. If, taking the behavior of rivals as given, moral hazard is important enough that it is optimal to have no profit-sharing, then the merger must be profitable. The divisions are strategically like independent firms, but they can now benefit from effort spillovers. By continuity, the same must be true even if there is a small amount of profit-sharing. If α > 1 is optimal, the merger should be profitable for strategic reasons alone. The merger provides a way to credibly commit to incentive schemes that are strategically advantageous but would not be credible absent the merger. A firm always would like to be able to commit to producing more than the Cournot-Nash equilibrium output, but if compensation schemes are unobservable or can be (secretly) renegotiated, this commitment is not credible. When effort spillovers are such that a merged firm finds it optimal (again, taking as given the behavior of outside firms) to have negative profit-sharing, then a merger serves as a credible commitment device.15 Because the merger, in this case, would increase output, it would probably also improve social welfare.16

5 Conclusion

This paper analyzes the behavior of a multiproduct firm when the assumption that the managers responsible for each product are perfect agents for the shareholders is relaxed. In particular, it shows that when the managers must exert unobservable effort to improve product quality, it is optimal for each manager to retain more than half of the profit her product generates, inducing incentives for intra-firm price competition. This explains why many multi-product firms have separate divisions that operate as independent profit centers. Because multiproduct firms may not always fully exercise its market power, there are important implications for antitrust policy. In fact, in a linear demand model, when product improving effort has positive spillovers (it improves the quality of the other product as well as the product of the manager that exerts the effort) the degree of profit-sharing can be small enough (possibly even zero or negative) that quality-adjusted price is lower when both products are under common ownership. In such cases, moral hazard improves

15This result has some parallels with Caillaud et. al. (1995). In this case, however, public announcements are not necessary. The commitment stems from the need to solve the moral hazard problem not from changing the agent’s reservation utility.

16I say probably because it is possible that the merger would exacerbate excessive effort incentives and this effect could outweigh the benefits from increased quantity.
social welfare and can make a merger that would have otherwise reduced consumer welfare beneficial for consumers.

While many, though not all, of the results in the paper were derived assuming linear demand, I suspect that qualitatively similar results would hold for many other demand specifications as well. Of course, establishing this conjecture would require simulations using different demand functions that are even less amenable to analytic analysis.

While the model in the paper assumed that managerial effort improved product quality, similar conclusions might sometimes apply when managerial effort is useful for cost reduction. Because costs are more readily observable, it will often be possible to condition the profit-sharing arrangement on costs and revenues separately. Forcing contracts, where one division gets all the profits unless costs are at or below the first best level, may be infeasible, however, due to the possibility of sabotage. As a result, the only feasible contracts may be contracts where each division gets a given share of its revenue and pays a (possible different) share of its costs. By giving the manager a large fraction of the benefits from cost reduction but a smaller fraction of her own revenue, the firm can decouple cost reduction incentives from price competition incentives. If, however, a manager can reduce costs not only by exerting effort but also by reducing quality, there will be an efficiency loss associated with this decoupling. Thus, moral hazard will still induce less revenue sharing than would be otherwise optimal, though probably to a lesser extent than when the unobservable effort affects quality rather than costs.
Appendix

Proof of Proposition 1  The argument in the text proves that so long as $e'(1/2) > 0$ the result holds. To show that $e'(1/2) > 0$, I totally differentiate (4) and (5) (when demand is given by $q(p^i - (e^i + \gamma e^{-i}), p^{-i} - (e^{-i} + \gamma e^i))$) with respect to $\alpha$ and solve for $e'(\alpha)$. Evaluating this at $\alpha = 1/2$ gives:

\[
e'(1/2) = \frac{2\{(q_1 + q_2)[(1 + \gamma)q - (1 - 3\gamma)(p - c)(q_1 - q_2)] + (p - c)(q_{11} + 2q_{12} + q_{22})[(1 + \gamma)q + 2\gamma(p - c)(q_1 - q_2)]\}}{(1 + \gamma)^2(q_1 + q_2)^2 + [4(q_1 + q_2) + 2(p - c)(q_{11} + 2q_{12} + q_{22})]k''}
\]

(21)

The denominator decreasing in $k''$ and is $-(1 + \gamma^2)(p - c)(q_1 + q_2)(q_{11} + 2q_{12} + q_{22})/2 < 0$ when $k''(e) = -(1 + \gamma^2)(q_1 + q_2)/4$.

The numerator is decreasing in $q$. From (4), we know that if $\alpha = 1/2$ then $q = (p - c)(q_1 + q_2)$. Substituting this into the numerator makes it the following:

\[
-2(p - c)\{(q_1 + q_2)[(1 - \gamma)q_1 + 2\gamma q_2]] + (p - c)(q_{11} + 2q_{12} + q_{22})[(1 - \gamma)q_1 + (1 + 3\gamma)q_2]\}
\]

(22)

This is increasing in $\gamma$. The $\gamma$ for which this is zero is:

\[
\gamma^* = \frac{(q_1 + q_2)[(p - c)X - q_1]}{(p - c)X(q_1 - 3q_2) - q_1^2 + q_1q_2 + 2q_2^2}
\]

(23)

Here, $X = -(q_{11} + 2q_{12} + q_{22}) > 0$. Since $\gamma^*$ is decreasing in $X$, the denominator is always negative whenever $\gamma \leq \frac{(q_1 + q_2)}{(q_1 - 3q_2)}$. Q.E.D.

Proof of Proposition 2  When $\tilde{k} > 1/2$ and $\gamma < .3$, the denominator of (16) is positive and the second order condition for $\alpha$ is always satisfied when the first order condition is. Thus, the direction of the effect of $\tilde{b}_2, \tilde{k},$ and $\gamma$ on the optimal $\alpha$ is determined by partially differentiating the numerator of the second line of (16) with respect to each of these parameters. Doing so with respect to $\tilde{k}$ gives:

\[
-2\tilde{k}_2^2(2\alpha - 1) < 0
\]

(24)
Partially differentiating this with respect to $\tilde{b}_2$ gives:

$$-\alpha[\gamma + 3\alpha(1 - \gamma) - 2\alpha^2(6 - \alpha)(1 - \gamma) + 10\alpha^3(1 - \gamma)^2] - 2\tilde{b}_2(2\alpha - 1)[2k + \alpha(1 - \gamma)(1 - \alpha(7 - \gamma) + 8\alpha^2(1 - \gamma) - \gamma)] + 3\tilde{b}_2^2\alpha(2\alpha - 1)^3(1 - \gamma)^2$$  \hspace{1cm} (25)

Now, I impose the first order condition by solving (16) for $\tilde{k}$:

$$\tilde{k} = \frac{2\alpha^3(1 - \gamma)(1 - \alpha(1 - \gamma)) - \alpha\tilde{b}_2[\gamma + 3\alpha(1 - \gamma) - 2\alpha^2(6 - \alpha)(1 - \gamma) + 10\alpha^3(1 - \gamma)^2] - \tilde{b}_2^2\alpha(2\alpha - 1)(1 - \gamma)[(1 - \alpha(7 - \gamma) + 8\alpha^2(1 - \gamma) - \gamma)] - \tilde{b}_2^2\alpha(2\alpha - 1)^3(1 - \gamma)^2}{2\tilde{b}_2^2(2\alpha - 1)}$$  \hspace{1cm} (26)

Substituting it into (25) gives the following:

$$\frac{1}{\tilde{b}_2}\{\alpha(4\alpha^2(1 - \gamma)(1 - \alpha(1 - \gamma)) + \tilde{b}_2[\gamma + 3\alpha(1 - \gamma) - 2\alpha^2(6 - \alpha)(1 - \gamma) + 10\alpha^3(1 - \gamma)^2] - \tilde{b}_2^2\alpha(2\alpha - 1)^3(1 - \gamma)^2\}$$  \hspace{1cm} (27)

It is not possible for (27) to be positive and the right hand side of (26) to be greater than 1/2. Thus, the optimal $\alpha$ is decreasing in $\tilde{b}_2$.

Partially differentiating the numerator of the second line of (16) with respect to $\gamma$ gives:

$$\alpha\{ -2\alpha^2(1 - 2\alpha(1 - \gamma)) + \tilde{b}_2[-1 + 3\alpha - 2\alpha^2(7 - 2\gamma) + 20\alpha^3(1 - \gamma)] + 2\tilde{b}_2^2(2\alpha - 1)[1 - \gamma - \alpha(4 - \gamma) - 8\alpha^2(1 - \gamma)] + 2\tilde{b}_3^2(2\alpha - 1)^3(1 - \gamma)\}$$  \hspace{1cm} (28)

This is decreasing in $\gamma$, so there is some $\hat{\gamma}$ so that (28) is positive if and only if $\gamma < \hat{\gamma}$. Q.E.D.

**Proof of Proposition 3**

(A) Evaluating the second fraction of (16) at $\alpha = 1$ and $\gamma = 0$, it becomes:

$$\frac{-\tilde{b}_2(1 + 2\tilde{b}_2(1 + \hat{k}) + \tilde{b}_2^2)}{(-1 + 4\hat{k} + \tilde{b}_2(2\hat{k} - 1))^3}$$  \hspace{1cm} (29)

This is always negative when $\hat{k} > 1/2$, so the optimal $\alpha$ must be less than one.

(B) Evaluating the second fraction of (16) at $\alpha = 1$ and solving for $\hat{k}$ gives:

$$\hat{k} = \frac{2\gamma(1 - \gamma) - \tilde{b}_2(1 - 8\gamma + 8\gamma^2) - 2\tilde{b}_2^2(1 - \gamma)(1 - 4\gamma) - \tilde{b}_3^2(1 - \gamma)^2}{2\tilde{b}_2^2}$$  \hspace{1cm} (30)
The right hand side of (30) is greater than $1/2$ if:

$$
\gamma < \frac{1 + 4b_2 + 5\tilde{b}_2^2 + \tilde{b}_2^3 - \sqrt{1 + 6b_2 + 12\tilde{b}_2^2 + 8\tilde{b}_2^3 - \tilde{b}_2^5}}{2 + 8b_2 + 8\tilde{b}_2^2 + \tilde{b}_2^3} \quad (31)
$$

The right hand side of (31) is less than .3 whenever $\tilde{b}_2 < .948$. Q.E.D.
References


